Math 515 Homework 2

Directions: NEATLY write all solutions on your own paper. Solutions should include details like what identities were used and any complicated computations. You may discuss the problems with other but write up your solutions on your own.

1) Suppose $f(x+0i) = e^x$, $f(0) = 1$ and $f(z)$ is entire. Show that if $f'(z) = f(z)$ then $f(z) = e^z$

2) Show that if $f(z)$ ananlytic on all of $\mathbb{C}$, then $f'(z_0) = \frac{1}{2\pi}\int_0^{2\pi} f(z_0 + e^{it})e^{-it}dt$ and **use it** show that $\int_0^{2\pi} \sin(e^{it})e^{-it}dt = 2\pi$.

3) Velocity fields of fluid flows are frequently given by complex functions with circulation defined as $circulation = Re(\oint_C \overline{f(z)})dz$. Compute the circulation for the velocity field of the fluid flow given by $f(z) = \frac{2+i}{z}$, where C is the circle of radius one centered at 0.

4)Evaluate $\frac{1}{2\pi i}\oint_C \frac{-2\left(z^2+1\right)\left(z^3-z^2-z+3\right)}{\left(z^2+2\,z+2\right)^4}\frac{\left(z^2+2\,z+2\right)^3}{\left(z^2+1\right)^2}dz$. Where $C = |z-(1/2+i)| = 1$ Hint: Compute the derivative of $\frac{\left(z^2+1\right)^2}{\left(z^2+2\,z+2\right)^3}$

5) Verify the limit from 5.22 $\lim_{R\to\infty}\frac{1}{\pi}\int_{\Gamma_R}\frac{\eta f(z)dz}{(z-\zeta)(z-\overline{\zeta})} \to 0$ if $|f(z)| \to 0$ for z in the upper half plane.