

Math 515 Homework 1

Directions: NEATLY write all solutions on your own paper. Solutions should include details like what identities were used and any computations. You may discuss the problems with other but write up your solutions on your own.

1) Show that for any two complex numbers z_1 and z_2

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$$

2) Show the principal branch cut of $\ln(z)$ is not continuous at $z = 1$, i.e. $0 \leq \theta < 2\pi$.

3) Show that for the solutions to $z^5 = 1$ sum to zero. What about $z^n = 1$ for any positive integer n ? You may use $1 + z + z^2 + z^3 + \cdots + z^k = \frac{1 - z^{k+1}}{1 - z}$

4) Show that the real and imaginary parts of $f(z) = z^2$ satisfy the PDE

$$r^2 \frac{\partial^2 \phi}{\partial r^2} + \frac{\partial^2 \phi}{\partial \theta^2} = -r \frac{\partial \phi}{\partial r}$$

5) Use this identity $1 + z + z^2 + z^3 + \cdots + z^n = \frac{1 - z^{n+1}}{1 - z}$ to verify the identity

$$\sin(\theta) + \sin(2\theta) + \sin(3\theta) + \cdots + \sin(n\theta) = \frac{\sin(\theta) + \sin(n\theta) - \sin((n+1)\theta)}{2 - 2\cos(\theta)}.$$

Hint: Plug in $z = e^{i\theta}$.