MATH 415/515 Quiz 4 Name:

Show all work. 5 points each.

1. What does the conformal map $w = z^2$ do to the first quadrant? Show with a graph where the points $1, 0, i$ in the z-plane are mapped to in the w-plane.

2.Solve the Dirchlet problem $\Delta\phi = 0$ on the first quadrant with boundary conditions $\phi(x, 0) = 1$ for $x > 0$ and $\phi(0, y) = 0$ for $y > 0$. Use the fact that the solution for the upper half plane with boundary condition $\phi(x, 0) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$ was $\phi(x, y) = 1 - \dfrac{\arctan(y/x)}{\pi}$

5 points extra credit: Use your solution to find the value on the ray $\theta = \dfrac{\pi}{4}$ and show the solution is independent of the distance form the origin.