

## Math 415/515 Homework 2

**Directions:** Students enrolled in 415 must complete 1-4. Students enrolled in 515 must complete 1-5. Scores will be prorated back to 25 for 415 students. NEATLY write all solutions on your own paper. Solutions should include details like what identities were used and any computations. You may discuss the problems with other but write up your solutions on your own.

1) Show that for any two complex numbers  $z_1$  and  $z_2$

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$$

2) Show the principal branch cut of  $\ln(z)$  is not continuous at  $z = 1 + 0i$ .

3) Show that for the solutions to  $z^5 = 1$  sum to zero. What about  $z^n = 1$  for any positive integer  $n$ ? You may use  $1 + z + z^2 + z^3 + \dots + z^k = \frac{1 - z^{k+1}}{1 - z}$

4) Show that the real and imaginary parts of  $f(z) = z^2$  satisfy the PDE

$$r^2 \frac{\partial^2 \phi}{\partial r^2} + \frac{\partial^2 \phi}{\partial \theta^2} = -r \frac{\partial \phi}{\partial r}$$

5) Show that the identity  $1 + z + z^2 + z^3 + \dots + z^n = \frac{1 - z^{n+1}}{1 - z}$  holds for any complex number  $z$ . Plug in  $z = e^{i\theta}$  and use this to verify the identity

$$\sin(\theta) + \sin(2\theta) + \sin(3\theta) + \dots + \sin(n\theta) = \frac{\sin(\theta) + \sin(n\theta) - \sin((n+1)\theta)}{2 - 2\cos(\theta)}.$$