Math 367 Review Test II

Be able to define the Fourier transform and inverse Fourier Transform for functions

$$\hat{f}(w) = \mathcal{F}(f(x))(w) = \int_{-\infty}^{\infty} f(x)e^{-2\pi i w x}dx \text{ and } \mathcal{F}^{-1}(\hat{f}(w))(x) = \int_{-\infty}^{\infty} \hat{f}(w)e^{2\pi i w x}dw$$

Be able to compute a simple fourier transform. Know the definition of convolution $f * g(x) = \int_{-\infty}^{\infty} f(x-t)g(t)\,dt$ for functions on defined the real line. Know these basic properties and be able to use them. Be able to derive 1,3,5,7

1) $\mathcal{F}(f(x)+g(x))(w) = \hat{f}(w) + \hat{g}(w)$ 2) $\mathcal{F}(cf(x))(w) = c\hat{f}(w)$

3) $\mathcal{F}(e^{2\pi i k x}f(x))(w) = \hat{f}(w-k)$ 4) $\mathcal{F}(f(x-k))(w) = e^{2\pi i k w}\hat{f}(w)$

5) $\mathcal{F}(f'(x))(w) = 2\pi i w\hat{f}(w)$ 6) $\mathcal{F}(xf(x))(w) = \frac{i}{2\pi}\frac{d}{dw}\hat{f}(w)$

7) $\mathcal{F}(f*g)(w) = \hat{f}(w)\cdot\hat{g}(w)$ 8) $\mathcal{F}(\mathcal{F}(f))(x) = f(-x)$

Be able to convert a Partial Differential equation into an ordinary one. Be able to state and prove Parseval's identity

Examples: I) Compute $\mathcal{F}(1_{[-1/2,1/2)}(x))(w)$ using the definition. Show $\mathcal{F}(f(ax))(w) = \frac{1}{a}\hat{f}(\frac{w}{a})$ for $a > 0$ and use these two to find $\mathcal{F}(1_{[-1,1)}(x))(w)$.

II) Compute $\mathcal{F}(e^{2\pi i 7x}1_{[-1/2,1/2)}(x))(w)$ using I and 3).

III) Convert the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ into an ODE.

Know the definitions of the DFT and the IDFT for sequences (also called vectors or signals) of length N. Be able to compute a simple DFT and IDFT. (say for $x = (1, 1, i, i)$)

$$X(k) = \mathcal{F}_N(x)(k) = \frac{1}{\sqrt{N}}\sum_{j=0}^{N-1} x(j)e^{2\pi i jk/N} \text{ and}$$

$$x(k) = \mathcal{F}_N^{-1}(X)(k) = \frac{1}{\sqrt{N}}\sum_{j=0}^{N-1} X(j)e^{-2\pi i jk/N}$$

Note I will not test directly over sampling ideas or discrete convolution but there may be some related ideas.