

## Math 367 Homework 4

**Directions:** NEATLY write all solutions on your own paper. You may discuss the problems with each other but you must write them up independently.

1) a) Use the Theorem about the DFT and convolution of sequences to show that  $x * y = y * x$  for two sequences of length  $N$ .

b) Find a sequence  $\{e(k)\}_{k=0}^{N-1}$  that is an identity for sequence convolution. That is,  $e * x = x$  for all sequences  $x$  of length  $N$ .

2) Edit the Maple code for Example 1 from the file FFT1 at the page (<http://people.uncw.edu/lammersm/courses.html>) to use sampling of the Gaussian  $f(x) = e^{-x^2}$  from  $[-1,1]$  to approximate the Fourier transform of  $f(x)$ . Does it look like a good approximation? What do you think is wrong? Extra credit: Edit the code further to give me a better approximation of the Fourier transform from sampling.

3) Let  $f$  be a piecewise continuous function of exponential order, .i.e. there exist  $a$  and  $M$  so that  $|f(t)| \leq Me^{at}$  for all  $t > 0$ . If  $\alpha$  is and  $\mathcal{L}(f) = F(s)$  a real number show

$$\mathcal{L}(e^{\alpha t} f(t))(s) = F(s - \alpha)$$

for  $s > a + \alpha$ . Use this to find  $\mathcal{L}^{-1}\left(\frac{1}{s^2 + 2s + 3}\right)$ . Hint  $\frac{1}{s^2 + 2s + 3} =$

$$\frac{1}{(s + 1)^2 + 2}$$

4) coming