Math 365 Homework 3 Due: Dec 6

Directions: NEATLY write all solutions on your own paper. Solutions should include details like " using integration by parts .." You may discuss the problems with others but write up your solutions on your own.

1) Let $B_1^n(0) = \{\mathbf{x} | \sum_{j=1}^n x_j^2 \leq 1\}$ be the unit ball centered at 0 in $\mathbb{R}^n$. Define the Volume of the ball of radius 1 in $\mathbb{R}^n$, β_n, by $\beta_n = \beta_{n-1} \int_{-1}^{1} (1-x^2)^{\frac{n-1}{2}} dx$. Similarly, let $S_1^n(0) = \{\mathbf{x} | \sum_{j=1}^{n+1} x_j^2 = 1\}$ be the unit sphere centered at 0 in $\mathbb{R}^{n+1}$.

a) Find β_1, β_2 and β_3 if $\beta_0 = 1$

b) If we let $c_n = \int_{-1}^{1} (1-x^2)^{\frac{n-1}{2}} dx$, then we have $\beta_n = \beta_{n-1} c_n$. Show that $c_0 = \pi$ $c_1 = 2$

c) If $c_n = \dfrac{n-1}{n} c_{n-2}$ (Prove this for 5 points extra credit) find the volume of $B_1^7(0)$.

d) The n-volume of the n dimensional sphere can be obtained by

$$vol_n(S_1^n) = (n+1)\beta_{n+1}. \text{ (see page 541)}$$

Show that for $n = 6$ you get the same answer by computing $vol_n(S_1^n) = \dfrac{2\pi^{(n+1)/2}}{\Gamma(\frac{n+1}{2})}$ where $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1}\, dt$.

Extra credit(5 pts): Find formulas for the Volume of the n dimensional Sphere and ball using $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1}\, dt$.