

Math 365 Homework 2

Directions: NEATLY write all solutions on your own paper. Solutions should include details like "by the triangle inequality we get .." You may discuss the problems with others but write up your solutions on your own.

1) Do one step of Newton's method to approximate a solution of

$$\begin{aligned}x + \cos(y) - 1.1 &= 0 \\x^2 - \sin(y) + .1 &= 0\end{aligned}$$

near $(0, 0)$. use a calculator or computer to find a solution. Do you think Newton's method converges?

2) Define the norm of a square matrix to be $\|A\| = \sup|\mathbf{Ax}|$ where $|\mathbf{x}| = 1$. (Def 2.9.6) Show $|\mathbf{Ab}| \leq \|A\| |\mathbf{b}|$

3) Apply Theorem 3.1.10 to determine for what c does the equation $x^4 + 2x^2 + y^3 - 3y = c$ determine a smooth manifold.

4) Let $\mathbf{z} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and consider the function $\mathbf{F} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z + x^2 + y^2 \\ \frac{5}{2} - x - y + 3z \end{pmatrix}$. Show that the \mathbf{z} so that $\mathbf{F}(\mathbf{z}) = \mathbf{0}$ is a smooth manifold and find a basis for the the tangent space at the point $\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$.