MATH 335 Review 3

Name
Student number

Show all work

1. (16 pts) True/False. Circle **T** or **F** (2 pts each). Justify your answer. (2 pts each)

T F a) The set of all vectors $\{\begin{bmatrix} a \\ b \end{bmatrix}, a+b=1\}$ is a subspace of $\mathbb{R}^2$.

T F b) The null space of an $m \times n$ vector is in $\mathbb{R}^m$

T F c) If T is a linear transformation that maps into $\mathbb{R}^4$ and dimension of the kernel of T is 2 then the range of T has dimension 2.

T F d) Every vector space of dimension 4 than it has exactly 4 differnet subspaces of dimension 1.

2. (12 pts) a) Define a subspace of a vector space.

b) Show why the set of polynomials $p(x) \geq 0$ are not a subspace of the $\mathbf{C}[0,1] = \{\text{continuous functions on } [0,1]\}$.

3. (12 pts) a) Define linearly independence for a vector space.

b) Explain why $\{\cos^2(x), \sin^2(x)\}$ is a linearly independent set in the vector space $\mathbf{C}[0, 1]$ while $\{\cos^2(x), \sin^2(x), 1\}$ is not.

4. (12 pts) a) Use the coordinate vectors to show that $\beta = \{1,\ 2t,\ -2 + 4t^2\}$ form a basis for $\mathbb{P}_3$.

b) Compute $[\mathbf{p}(t)]_\beta$ for $\mathbf{p}(t) = 1 + t^2$.

5. (12 pts) State the Rank Theorem. If A is a 7×5 matrix what is the largest possible rank of A and A^T. **EXPLAIN**

6. (12 pts) Find bases for Col(A) and Row(A^T) and Null(A) if

$$A = \begin{bmatrix} 7 & 5 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7. (12 pts) Let A be a $m \times n$ matrix. Show the equation $A\mathbf{x} = \mathbf{b}$ has a solution for all $\mathbf{b} \in \mathbb{R}^m$ iff the equation $A^T\mathbf{x} = \mathbf{0}$ has only the trivial solution.

8. (12 pts) Find the change of coordinate basis from $\mathcal{B}$ to $\mathcal{C}$ where $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$, $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ and $\mathbf{b}_1 = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$, $\mathbf{c}_1 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$, $\mathbf{c}_2 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$