Optimization Trade-Offs in the Design of Wireless Sensor and Actor Networks

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***Abstract*—Wireless sensor and actor networks (WSANs) are composed of static sensor nodes and mobile actor nodes. We assume actors have a random initial location in the two-dimensional sensing area. The objective is to move each actor to a location such that every sensor node is within a bounded number of hops from some actor. Because sensor nodes have limited energy, the new actor locations are chosen as to minimize the transmission range required from the sensor nodes. However, actors also have a limited (although larger) power supply, and their movement depletes their resources. It follows that by carefully choosing the new actor locations, the total actor movement can be minimized. In this paper, we study the trade-off between minimizing sensor transmission radius and minimizing actor movement. Due to the complexity of the problem, we introduce an optimal ILP formulation, and compare its results against a proposed heuristic. For the ILP solution to be feasible, we introduce a finite set of potential actor positions such that an optimal solution is guaranteed to be found within this set.**

I. Introduction

A wireless sensor and actor networks (WSANs) consists of two groups of devices. The first is a collection of sensor nodes. Sensors typically have a fixed location, and their resources are limited, in particular, their battery life. The second is a collection of actor nodes. These have significantly more resources than the sensor nodes, and are often mobile. When a sensor node gathers data about some event, it sends the sensing information to an actor. Actors make decisions for various issues and execute necessary actions based on the received information from sensor nodes and from other actors [1]. WSANs can be used in numerous applications, such as battlefield surveillance, urban search and rescue, environmental monitoring, etc..

The effectiveness and performance of the network depend heavily on the position of the actor nodes. For example, a proper placement of actors can improve network lifetime by reducing the transmission range required for sensor nodes, which in turn increases the lifetime of their batteries. Similarly, proper placement of actors can reduce the number of hops traversed between a sensor and its closest actor, which in turn affects network delay and also affects network lifetime. Thus, given the fixed locations of a set of sensor nodes, and given the initial locations of a set of mobile actor nodes, we focus on choosing new actor locations that satisfy certain performance requirements, as discussed further below.

A factor to consider is whether communication between sensors and actors is via a single hop or a multi-hop approach. In the single-hop approach, each sensor is within the communication range of at least one actor node. In the multi-hop approach, data from some sensors can only reach an actor node by traversing other sensor nodes. The multi-hop approach is likely to consume less energy than the single hop approach, because multiple short transmissions consume less energy than a single long-range transmission. However, the number of hops between a sensor node and an actor play an important role due to data latency [2], because latency of data is often proportional to the number of hops [3]. Thus, a bounded number of hops is desirable.

From the above observations, our first objective is to determine the smallest transmission radius, r_{min}, such that each sensor node can communicate with at least one actor within a maximum of d hops, where d is a parameter to our problem. Therefore, if there are k actors, k locations must be determined such that all sensor nodes can reach one of these locations within d sensor hops.

Although the actors are equipped with more powerful resources when compared to sensor nodes, we assume their energy is nonetheless limited, such as from a battery or an on-board fuel supply. We assume the movement of an actor consumes significantly more energy than computation and the collection of data from sensors [4]. Because actors cannot be easily recharged after their deployment, minimizing their movement is a critical issue in WSANs [5].

Our refined goal is thus as follows. Given is a set of n fixed sensor nodes, and k mobile actor nodes, both of which are randomly located in an open field. We seek the joint optimization of two values. As mentioned above, the first one is minimizing the transmission radius such that each sensor can communicate with at least one actor within d hops. The second one is minimizing the total actor movement from the initial locations of the actors to their newly chosen locations. In this paper, we introduce this problem as M^{2}RAMS (Minimizing Multi-hop Range and Actor Movement Simultaneously). Due to the complexity of the problem, we present a heuristic solution. For small instances of the problem, we also compare the heuristic against an optimal ILP solution.

Given that two values are to be optimized, that is, the minimum transmission radius and the total actor movement, there are two alternative approaches that we could apply to the heuristic and to the ILP. The first is the *double-step approach*, in which we first minimize the transmission range, and then, after choosing the actor locations, we minimize the actor movement by pairing actors with their new locations. The second is the *single-step approach*, in which each actor location and its corresponding actor are chosen together. We show that the single-step approach outperforms the double-step approach.

In our earlier work [7], we presented a single-hop version of M^{2}RAMS (simply known as MRAMS). This paper differs from our earlier work in two significant respects. First, we extend both the heuristics and the ILP formulation to address the multi-hop sensor network. Second, and most importantly, the finite set of potential actor positions used in [7] differs from the ones used in this paper. In [7], the potential actor positions were borrowed from [2][6], and, although they guarantee that the ILP solution will find an optimal transmission radius, they do not guarantee an optimal movement of actors. Here, we present a novel alternative set of potential positions that guarantee an optimal solution is found for both transmission radius and for actor movement.

II. Problem Statement and Groundwork

We define the problem more formally as follows. We are given a set S with the locations of n sensors, $s_1, \ldots s_n$, which are randomly deployed in a two-dimensional plane. Also given is a set A of the locations of k actors, $a_1, \ldots a_k$, that are also randomly placed on the field. An *actor placement* $\mathcal{F}$ is a function that assigns a new location to each actor. That is, $\mathcal{F}(a_j)$ returns the new location of the actor whose initial location was a_j. An upper bound d is also given on the allowed hop count from any sensor node to its closest actor once actors have been relocated.

Let $min\text{-}hops(S, A, \mathcal{F}, r)$ be the minimum number of hops from any sensor node to any actor assuming each actor and sensor have a transmission range of r. Let $min\text{-}radius(S, A, \mathcal{F}, d)$ be the smallest value of r such that $min\text{-}hops(S, A, \mathcal{F}, r) \leq d$. We earlier referred to this value as r_{min}. Finally, let $total\text{-}move(A, \mathcal{F})$ be the sum of the distances that actors must traverse to move to their new locations, i.e.,

$$total\text{-}move(A, \mathcal{F}) = \sum_{1 \leq i \leq k} distance(a_i, \mathcal{F}(a_i))$$

Then, $\mathcal{F}$ is said to be a *solution* iff, for any actor placement $\mathcal{F}'$, either

$$min\text{-}radius(S, A, \mathcal{F}, d) < min\text{-}radius(S, A, \mathcal{F}', d)$$

or

$$min\text{-}radius(S, A, \mathcal{F}, d) = min\text{-}radius(S, A, \mathcal{F}', d) \wedge \\ total\text{-}move(A, \mathcal{F}) < total\text{-}move(A, \mathcal{F}')$$

We next overview some of the background work needed to solve this problem.

A. Minimizing Transmission Range

Consider focusing on finding the value of r_{min} when $d = 1$, i.e., finding locations for the actors such that all sensor nodes are within a distance r_{min} from at least one actor. This sub-problem of M^{2}RAMS is equivalent to the well-known Euclidean p-center problem [8][9][10], with sensors corresponding to demand points, and actors corresponding to supply points. We next overview the typical solution to this problem [11].

Finding a solution is non-trivial because there are an infinite number of locations where actors may be placed (any point on the plane), and also an infinite number of radii r to consider, since we do not assume r is discrete. Although NP-hard, the problem is NP-complete [12], and a solution can be found by carefully selecting a finite set of possible actor locations and a finite set of radii. It can be shown [6][9] that the optimum radius r_{min} must belong to a finite set $R(S)$, where $|R(S)| \in$ O(n^3). This is because in the solution there must be at least one subset of sensors that is covered exactly by a circle of radius r_{min}, i.e., the circle contains sensors in its periphery. The number of minimum circles with sensors in the periphery is bounded, and has shown to be in O(n^3).

To solve the problem, assume there exists a procedure, $cover(S, k, r)$, to determine if, for a given radius r, it is possible to cover all sensor nodes with k actors. Obviously, if $r \geq r'$ and $cover(S, k, r')$ is successful, then so will $cover(S, k, r)$. Hence, a binary search is performed over the elements of $R(S)$ to find the smallest r satisfying $cover(S, k, r)$. This yields the optimum radius r_{min}.

The complexity of the problem arises not from the binary search ($O(\log n)$ steps), but from performing procedure $cover(S, k, r)$, which in itself is an NP-complete problem. This is solved by observing that, for a given radius r, if $cover(S, k, r)$ is possible, then the actor locations can be chosen from a finite canonical set of points $P(S, r)$, where $|P(S, r)| \in$ O(n^2) [2][8][13]. Thus, $cover(S, k, r)$ may be implemented by testing all subsets of $P(S, r)$ of cardinality k, which has exponential complexity.

The canonical set $P(S, r)$ is obtained as follows. [2][8][13]. Consider Fig. 1. (a), where a subset of sensor nodes is covered by a circle of radius r. If no sensor is at the boundary of the circle, the circle can be moved in any direction until a sensor is reached (Fig. 1. (b)). Then, the circle can be rotated around the sensor until a second sensor reaches the boundary (Fig. 1. (c)). Note that the same subset of nodes remains covered after these two steps. Thus, $P(S, r)$ consists of the centers of all circles of radius r with (at least) a pair of sensors at their periphery. Note also that, for every pair of points on the plane, there can be only two circles of radius r that touch both points. Thus, $|P(S, r)| = 2 \cdot \binom{n}{2} \in$ O(n^2).

B. Radius List for Multi-hop Communication

As we described in section II-A, the desired minimum radius, r_{min}, is contained within a set $R(S)$ of cardinality O(n^3). This set is computed for the transmission between an actor and a sensor node, but it does not include the

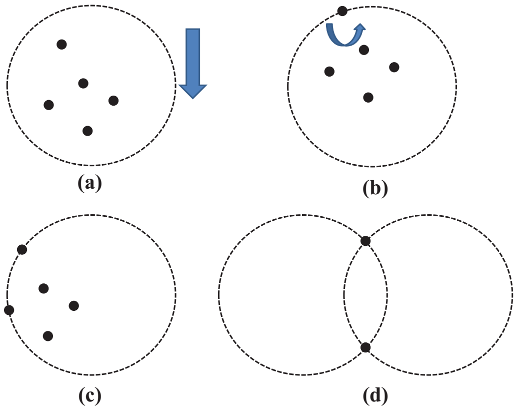

Fig. 1. Canonical potential positions for actors

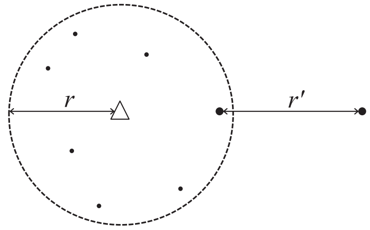

Fig. 2. Additional radius for multihop communication.

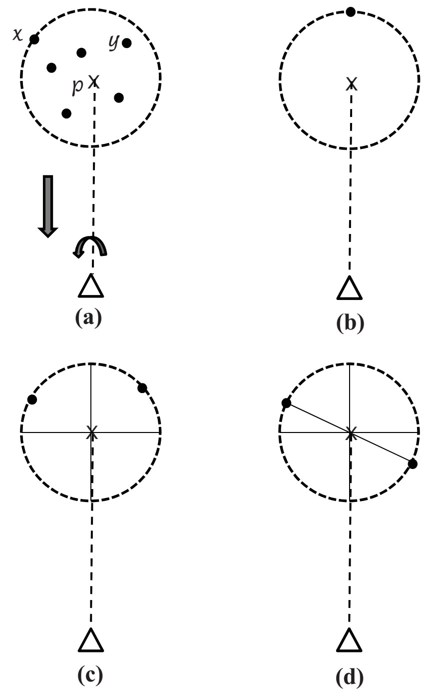

Fig. 3. Improved potential positions for actors

transmissions between sensors along the path to an actor. Let's consider for example Fig. 2, where we have a single actor and a hop bound of two. The distance r' between the two sensor nodes on the right is larger than the distance between the actor and its first-hop sensors. Hence, r_{min} must be at least r'. To account for this [11], we must use a superset $R_M(S)$ of $R(S)$ that also includes the distance between every pair of sensor nodes. Note that $R_M(S)$ remains $\mathrm{O}(n^3)$, so the binary search to compute r_{min} is not seriously affected.

As argued in [14], the same set of actor positions for single hop can be used for multi-hop. Thus, we let $cover_M(S, k, r, d)$ represent whether with a radius r we can find k actor positions such that a sensor can reach some actor among k actors within d hops.

III. Minimizing Actor Movement

There have been several works in the area of minimizing actor movements in WSANs. Most works address only the selection of locations where actors (or other significant nodes) are to be placed. Some of these works are the following. In [2], authors defined k-sink placement problem whose objective is to minimize the maximum hop-distance between sensor node and its nearest sink. Also, the placement of actors for load balancing is addressed. Their objective is to find clusters such that the size of each cluster is bounded and the number of hops from each sensor to each cluster is also bounded [15]. In [3], in order to reduce data latency, authors proposed two approaches based on genetic algorithms, which show how to choose locations for multiple sinks such that the average hop and euclidean distance between all sensor nodes and their nearest sink are minimized. In [16], they presented an actor positioning scheme that provides both maximizing coverage of area and minimizing data gathering latency.

Once locations are chosen for actors, pairing actors with their new location poses an optimization problem if the distance traveled by actors is to be minimized. In [17], the authors proposed a pairing of actors and cluster heads using heuristic based on matching theory. Its goal is to minimize total actor movement or total matching distance between actors and cluster head. On the other hand, for minimizing of traveling distance, [18] studied for different goal to restore connectivity among actors by relocating actors with minimal movement in case of actor failure.

Our earlier works [7], [11] distinguish themselves from the above works in two ways. First, the transmission radius, instead of a constant, is a parameter to be optimized. Second, we considered a joint optimization of the transmission radius and the actor movement. Note that minimizing sensor transmission range and actor movement are conflicting goals. E.g., actor movement is minimized to zero by simply extending the transmission range.

Our earlier work, however, is based on the canonical potential positions $P(S, r)$ which are given in [2][8][13]. Although an exhaustive search of these potential positions ensures that the minimal radius is found, they do not guarantee a minimum amount of movement from the actors. Therefore, below we propose an alternative new set of potential positions $P_M(S, A, r)$, from which a minimal actor movement is guaranteed to be found.

Consider Fig. 3. (a). It consists of an actor (the small triangle) in its original position, and the dashed circle corresponds to the area of radius r that the actor will cover after it has moved to its new position. The dots represent sensor nodes that will be associated with this actor once it reaches its new position. Assuming that the sensor nodes covered by the actor are somehow previously identified, we would like to determine which new position would minimize the movement of the actor.

In Fig. 3. (a), it is obvious that the new position is not optimal, for the following reason. If the actor turns slightly counterclockwise before beginning its movement forward, it is able to cover the same set of sensor nodes by traveling a slightly shorter distance. To visualize this, assume the path of the actor (the dashed line) and its coverage area (dashed circle) is a pendulum, whose fulcrum is at the current position of the actor. If the pendulum turns clockwise, then, sensor x, who is on the left border, will be outside of the actor range. However, if the pendulum turns slightly counterclockwise, then all sensors remain in the coverage area. In particular, x is now farther inside the coverage area, rather than at its border. The length of the arm of the pendulum can then be made shorter until some node touches the border of the coverage area (most likely x). Since the arm of the pendulum is shorter, the actor has to travel a shorter distance than before.

Any optimal actor position cannot be improved by performing the above steps (rotation followed by shortening the distance). We perform a case analysis to identify these optimal positions.

A. One Sensor at the Border

If there are no sensor nodes at the border of the coverage area, then, similar to the above, the actor position is not optimal.

On the other hand, if there is a single sensor on the border of the coverage area, then it is easy to show that the only place that prevents a shorter distance for the sensor is directly along the path of the actor, as shown in Fig. 3. (b). This creates a total of $k \cdot n$ potential positions, one for every sensor and actor pair, to be included in our set $P_M(S, A, r)$.

B. Two Sensors at the Border

Let us suppose that next that there are a pair of sensors at the border of the coverage area, and assume we divide the area into quadrants, as shown in Fig. 3. (b). Assume both sensors are on the same quadrant (not shown in the figure), then irrespective of the quadrant, the rotate and shortening method shows that the position is not optimal. On the other hand, assume they are in different quadrants. We first address if they are in adjacent quadrants, followed by non-adjacent quadrants.

For the adjacent quadrants, we have four choices. If the sensors are in the upper left and lower left quadrant, then a counter-clockwise rotate followed by a shortening shows the position is not optimal. The same argument applies for the right quadrants. If the sensors occur in the lower quadrants, no rotation is necessary; the distance can simply be reduced. On the other hand, if they occur in the upper quadrants, as shown in Fig. 3. (b), then any rotation would leave one sensor outside of the coverage area. Thus, this is a possible optimal position.

Consider finally when the sensors are in non-adjacent quadrants, as shown in Fig. 3. (c). It can be shown [14] that the only potential optimal position is when the sensors are along a diameter of the coverage area (i.e., $2 \cdot r$ away from each other). This adds at most $\mathrm{O}(k \cdot n^2)$ potential positions to our set $P_M(S, A, r)$.

C. Three or More Sensors at the Border

Finally, assume there are three or more sensors at the border of the coverage area. Three or more points on the plane define a single circle. The circle must be of radius r, i.e., the coverage area of the actor. It is unlikely that many triples of sensor nodes will precisely define a circle of radius r. Hence, rather than trying to eliminate them from our set $P_M(S, A, r)$ by a complex case analysis, we simply add them to our set, because they will not significantly influence the time required to find an optimal solution.

IV. ILP Formulation for M^2RAMS

In this section, we present an ILP formulation of the M^2RAMS problem to quantify the effect of choosing the above optimal set (with respect to movement) $P_M(S, A, r)$ of actor positions vs. the canonical set $P(S, r)$ given in [2][8][13]. In Section VI, we also represent various results based on our ILP formulation.

The ILP below is given a set of sensors S, a set of actors A, a radius r, a hop count d, and a set of potential positions P. The ILP will determine if $cover_M(S, |A|, r, d)$ is possible, and if so, it will return an actor placement function $\mathcal{F}$ that yields the minimal movement for the actors.

Note that a binary search over all values in $R_M(S)$ is still necessary, since the ILP will only determine if a specific radius r is sufficient to cover all sensors, but does not directly obtain r_{min}. Note also that both choices for P, i.e., $P_M(S, A, r)$ of actor positions vs. the canonical set $P(S, r)$, will yield the same minimum radius r_{min}. However, $P_M(S, A, r)$ guarantees the optimal actor movement.

A. Notation

We define notations in the ILP formulation as follows.
P: set of potential locations for the actors.
n: total number of sensor nodes.
m: total number of potential locations for actors ($m = |P|$).
k: total number of deployed actors.
i: index for a sensor node ($1 \le i \le n$).
j: index for a deployed actor ($1 \le j \le k$).

p: index for a potential position in the set P $(1 \leq p \leq m)$.
$\lambda_{j,p}$: distance from potential position p to initial location of actor j.
$sensor\text{-}hops(i, d)$: potential positions within d-hops from sensor node i.

Also, we define the following integer variables.

$$Y_{j,p} \quad = \quad \begin{cases} 1, & \text{if an actor } j \text{ moves to potential position } p \\ 0, & \text{otherwise.} \end{cases}$$

$$Z_p \quad = \quad \begin{cases} 1, & \text{if potential position } p \text{ is chosen as} \\ & \text{one of the } k \text{ positions} \\ 0, & \text{otherwise.} \end{cases}$$

B. ILP formulation

Our objective function is to minimize the sum of the distances between the initial and the final positions for the actors. So, the objective function is as follows.

$$\textbf{Minimize} \sum_{j=1}^{k} \sum_{p=1}^{m} \lambda_{j,p} \cdot Y_{j,p} \tag{1}$$

Subject to:

$$\sum_{p=1}^{m} Y_{j,p} \leq 1, (\forall j) \tag{2}$$

$$Y_{j,p} \leq Z_p, (\forall j, \forall p) \tag{3}$$

$$\sum_{p=1}^{m} Z_p \leq k \tag{4}$$

$$\sum_{p \in sensor\text{-}hops(i,d)} Z_p \geq 1, (\forall i) \tag{5}$$

C. Justification of the ILP equations

The objective function, given in (1), minimizes the total distance between initial actor positions and their corresponding final potential positions, such that constraints (2) through (5) are satisfied. From constraint (2), each actor j is allowed to move to at most one potential position p. Constraint (3) requires that if an actor selects some potential position, that position must be among the k selected positions. Constraint (4) forces the number of selected actor locations for M^{2}RAMS to be k. Finally, constraint (5) requires that each sensor be within at most d hops from some actor.

D. Numerical Results

As the first performance analysis in our contribution, we evaluate the effectiveness of the new potential positions $P_M(S, A, r)$ vs. the previous set $P(S, r)$ by generating random networks and applying the ILP formulation to them using the CPLEX [19] solver. The networks consisted of a square-shaped sensor area of size 500 $\times$ 500 m^2. Furthermore, each point in our graphs represents the average of 10 different graphs where the sensors and actors are randomly deployed. We implemented our simulations using 50 sensor nodes and hop bounds of d = 1 and d = 2. The number of actors ranges between 4 and 10.

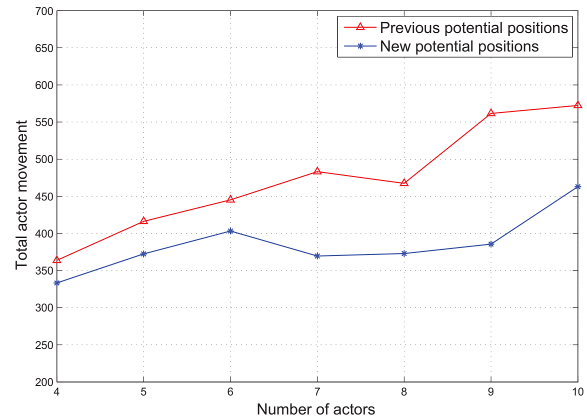

Fig. 4. Comparison of the total actor movement with 50 sensor nodes by *single-ILP* with hop bound $d = 1$ using previous potential positions and new potential positions.

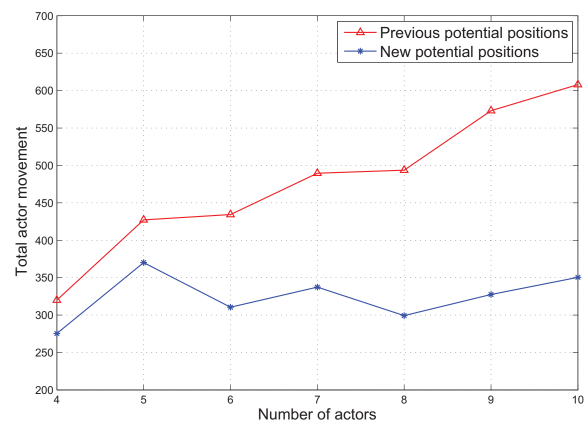

Fig. 5. Comparison of the total actor movement with 50 sensor nodes by *single-ILP* with hop bound $d = 2$ using previous potential positions and new potential positions.

Fig. 4 and 5 show total actor movement (with same minimum radius r_{min}) between the new potential positions $P_M(S, A, r)$ and the previous set $P(S, A, r)$. We can verify that there is a clear advantage of the new potential positions by reducing actor movement considerably in M^{2}RAMS.

V. Heuristics for M^{2}RAMS

Solving M^{2}RAMS requires us to check if all sensor nodes can be covered by at least one actor with a bounded number of hops, i.e., solving $cover_M(S, |A|, r, d)$. Once the smallest value of r is found (through binary search), a mapping function $\mathcal{F}$ from actors to potential positions $P_M(S, A, r)$ is found. Solving $cover_M(S, |A|, r, d)$ and obtaining $\mathcal{F}$ are both NP-complete problems.

We have previously evaluated multiple heuristics for $cover_M(S, |A|, r, d)$ with single-hop $(d = 1)$ and multi-hop

($d > 1$) scenarios [11], [13], and discovered that a greedy heuristic, described below, works best for finding the smallest radius r_{min}.

On the other hand, for movement, we have two options: (i) the *double-step approach*, in which $cover_M(S, |A|, r, d)$ is solved first, and $\mathcal{F}$ is chosen to map actors only to the locations found in the first step, and (ii) the *single-step approach*, in which $cover_M(S, |A|, r, d)$ and $\mathcal{F}$ are solved jointly. In a sense, two heuristics have to operate concurrently, one for $cover_M(S, |A|, r, d)$, and one for $\mathcal{F}$. We have presented in [7] *single-step heuristic* for a single-hop network. Compared to the *double-step approach*, it delivers significant gains in actor mobility, at the expense of only a modest increase in transmission radius.

We would like to investigate the performance of this *single-step heuristic* applied to a multi-hop network, and also using the new locations $P_M(S, A, r)$ introduced in this paper. As an introduction, we first discuss our heuristic for $cover_M(S, |A|, r, d)$, and then we explain how to merge this with a heuristic for $\mathcal{F}$, to obtain a *single-step heuristic*.

To solve for $cover_M(S, |A|, r, d)$, actor locations are iteratively chosen one by one. An actor location is selected if it covers more sensors than any other actor locations. Once the location is chosen, the sensors it covers are removed from the set of available sensors, and the process is repeated until all sensors are covered.

However, to incorporate actor movement, i.e., to incorporate $\mathcal{F}$, we need more flexibility in the choice of the next actor location. At each iteration, we select a subset of actor locations that provide sensible, but not necessarily the largest, coverage of sensors. From this set, we chose the location with the smallest distance to any unassigned actor. This adds an (actor, new location) pair to $\mathcal{F}$. The flexibility in choosing the subset of actor locations is governed by a user-defined parameter α, $0 \leq \alpha \leq 1$.

We thus perform the following steps for k iterations.

- Determine the maximum number of sensor nodes (*MaxSensors*), which are covered by any one potential position from $P_M(S, A, r)$.
- Find the potential positions from $P_M(S, A, r)$ that cover at least (*MaxSensors* $\times$ $(1 - \alpha)$) sensor nodes.
- Among these potential positions, select the pair (initial actor position a, potential position p) with the closest distance from a to p. Add this pair to $\mathcal{F}$.
- Search for sensor nodes within d hops from p.
- The above sensor nodes are then removed from the graph.

The heuristic accepts r iff all sensor nodes are removed from the graph after k iterations. The pseudocode of the heuristic is presented in more detail in Algorithm 1.

Note that, because at each iteration we do not choose the location with the best coverage, we might end up with a larger radius than the *double-step heuristic* in [13] (i.e., if we choose $\alpha = 0$). However, we would like to emphasize that significant gains are made in movement (as shown in Section VI) without significant sacrifices in radius.

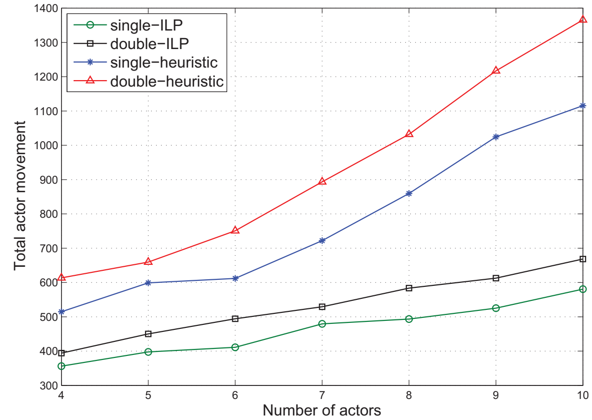

Fig. 6. Comparison of the total actor movement with the number of sensor nodes = 50 and α = 0.1 by different approaches with hop bound = 1

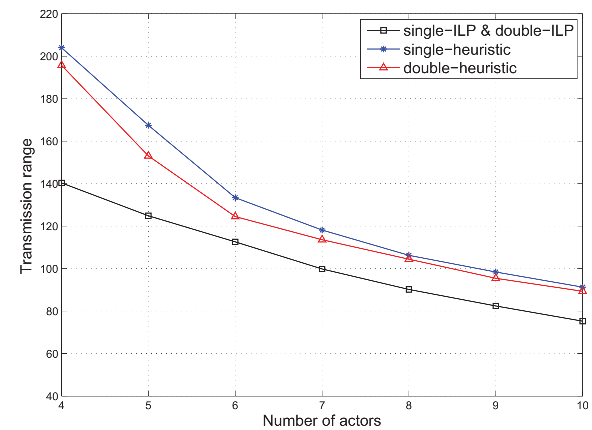

Fig. 7. Comparison of the transmission range with the number of sensor nodes = 50 and α = 0.1 by by different approaches with hop bound = 1

VI. Experimental Evaluation

In this section, we analyze the performance of our proposed heuristic and ILP formulation. We define four different approaches: *single-heuristic*, *double-heuristic*, *single-ILP* and *double-ILP*.

Our heuristic and ILP formulation in Sections V and IV are referred to as *single-heuristic* and *single-ILP*, respectively.

On the other hand, *double-ILP* first finds a solution to $cover_M(S, k, r, d)$ using the same constraints as in Section IV, but without an objective function (i.e., actor movement) to be minimized. Then, the placement function $\mathcal{F}$ is found by a second run of the ILP with set P containing only the actor positions found in the first step. Finally, *double-heuristic* first solves $cover_M(S, k, r, d)$ via Algorithm 1, but without lines 6, 7, and 8 (i.e., without regard for movement), and then it obtains $\mathcal{F}$ by applying a greedy algorithm to match actors with their new positions.

As in Section IV-D, we simulated our various experiments in a square-shaped sensor area of size $500 \times 500\ m^2$ where n

Algorithm 1 Inputs: S, A, r, d, α, Output: *success* or *failure*

Set mapping $\mathcal{F} \leftarrow \emptyset$
Set unmapped actor positions $A' \leftarrow A$
Set uncovered sensor nodes $S' \leftarrow S$
while $|A'| > 0$ **do**
  Find the maximum number of sensor nodes, *MaxSensors*, from S', which are covered by a single potential position in $P_M(S, A, r)$ using at most d hops.
  Let T be the subset of $P_M(S, A, r)$ such that each position in T covers at least (*MaxSensors* $\times$ $(1-\alpha)$) sensor nodes in S' using at most d hops.
  Let (a, p), where $a \in A'$ and $p \in T$, be the pair with minimum distance from a to p.
  Set $F \leftarrow \mathcal{F} \bigcup (a, p)$
  Set $A' \leftarrow A' - \{a\}$
  Set $S'_d \leftarrow$ subset of S' within d hops of p
  Set $S' \leftarrow S' - S'_d$
end while
Return *success* if $S' = \emptyset$, otherwise *failure*.

sensor nodes and k actors are randomly deployed in the area. Each experiment represents the average result of ten different graphs. When we execute *single-heuristic* and *double-heuristic* for M^{2}RAMS, we used a custom-made program, and the results for *single-ILP* and *double-ILP* are obtained using CPLEX [19].

In our first group of simulations, we compare all four different methods using the new set $P_M(S, A, r)$ of actor positions. We use two evaluation criteria: the minimum radius obtained, and the total actor movement. Due to the overhead of the ILP, we limit the number of sensor nodes to 50. We choose two values for α, 0.1 and 0.3, and consider networks with hop bound d = 1 and d = 2. The number of actors ranges from 4 to 10. The results of the first simulations are shown in Fig. 6, 7, 8 and 9.

As expected, *single-ILP* always outperforms the other schemes because it is guaranteed to be optimal. The next best scheme is *double-ILP*, which is optimal for transmission radius, but not optimal for actor movement. Nonetheless, it is able to outperform the heuristics. When we analyze performance of the heuristics, *single-heuristic* is sufficiently better for total movement distance than *double-heuristic* as the number of actors increases. In particular, it is observed that the total actor movement for the *single-heuristic* is at most twice the total actor movement of the *single-ILP*. Reaching a value no more than twice the optimal is a significant achievement for a heuristic.

For transmission range, as the number of actors in the network grows, the result of our *single-heuristic* becomes closer to the optimum using ILP. For example, when the number of actors is 10, the gap between the *single-ILP* and *single-heuristic* is very small.

Lastly, we evaluate the impact of the parameter α on our *single-heuristic*. Since we are not constrained by the overhead

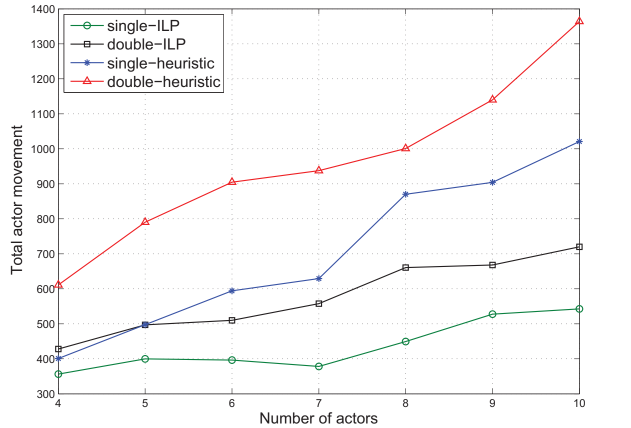

Fig. 8. Comparison of the total actor movement with the number of sensor nodes = 50 and α = 0.3 by different approaches with hop bound = 2

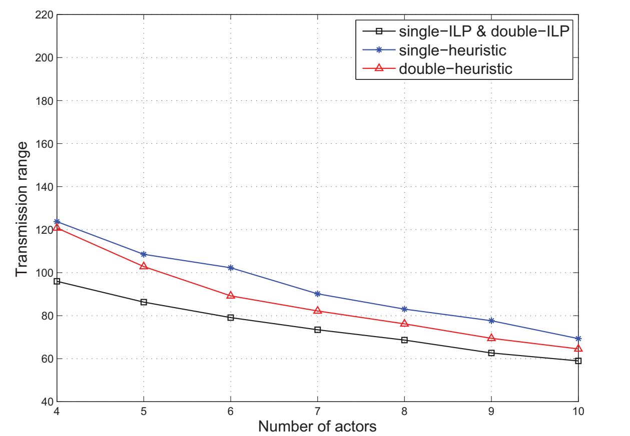

Fig. 9. Comparison of the transmission range with the number of sensor nodes = 50 and α = 0.3 by by different approaches with hop bound = 2

of the ILP, we increase the number of sensor nodes to 100 and show average results by 30 different graphs. We choose α = 0.0, α = 0.1 and α = 0.2, and consider hop bound as d = 1, d = 2 and d = 3 respectively. The results are shown in Fig. 10 through 15.

The figures show that the total actor movement decreases significantly as α increases. This is because of the greater flexibility in choosing actor positions at each step in the heuristic. With respect to transmission range, only a slight increase in transmission range occurs as we increase α. Thus, a significant decrease in actor movement is obtained at the cost of a small increase in transmission range.

Note also that, for hop bound d = 1 case, as the number of actors increase, the difference between the transmission range of the three α values decrease. For hop bound d = 2 and d = 3 cases, the difference in transmission radius of the three α parameters is negligible.

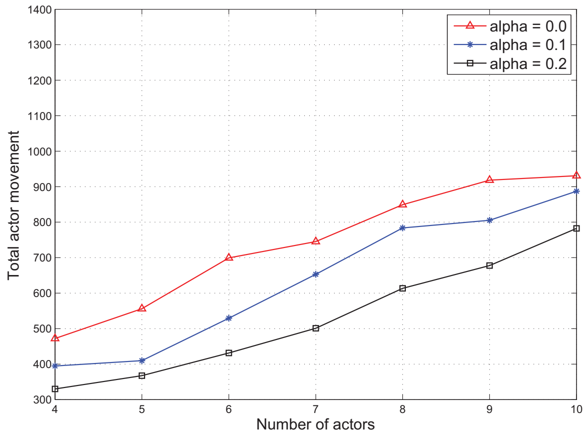

Fig. 10. Total actor movement with the number of sensor nodes = 100 by different α(alpha) using *single-heuristic* with using hop bound = 1

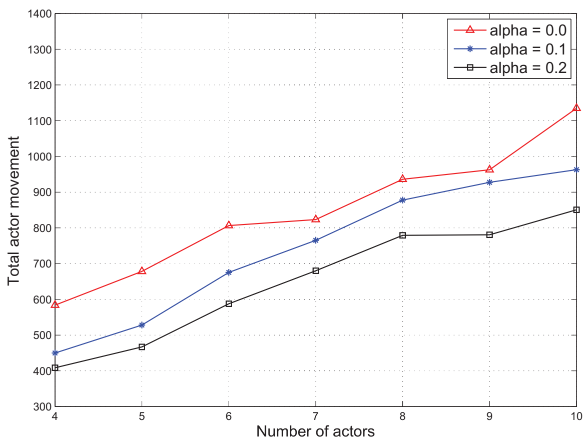

Fig. 11. Total actor movement with the number of sensor nodes = 100 by different α(alpha) using *single-heuristic* with using hop bound = 2

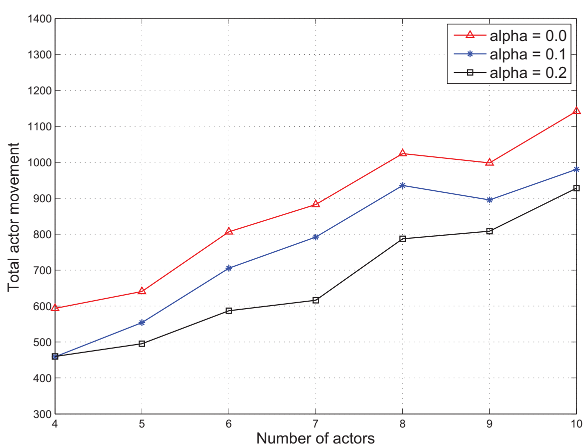

Fig. 12. Total actor movement with the number of sensor nodes = 100 by different α(alpha) using *single-heuristic* with using hop bound = 3

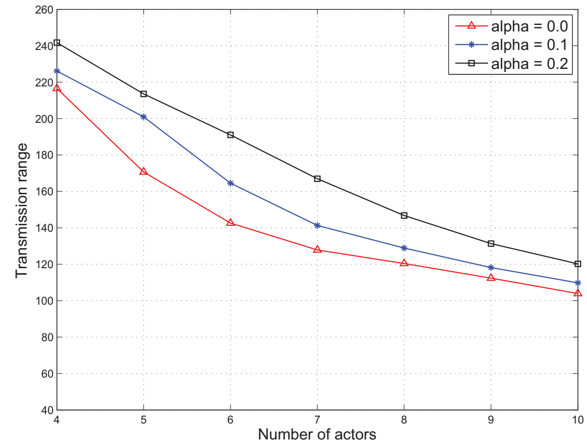

Fig. 13. Transmission range with the number of sensor nodes = 100 by different α(alpha) using *single-heuristic* with using hop bound = 1

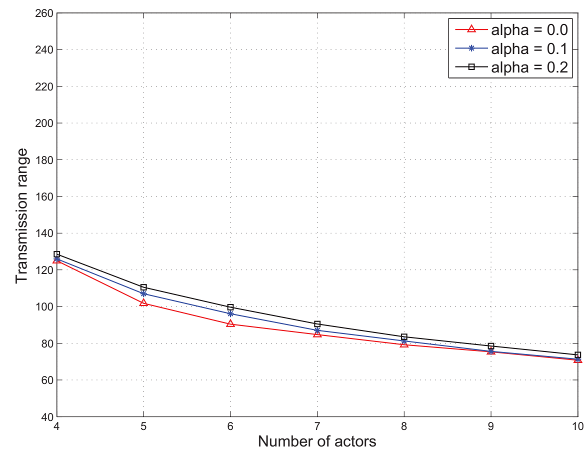

Fig. 14. Transmission range with the number of sensor nodes = 100 by different α(alpha) using *single-heuristic* with using hop bound = 2

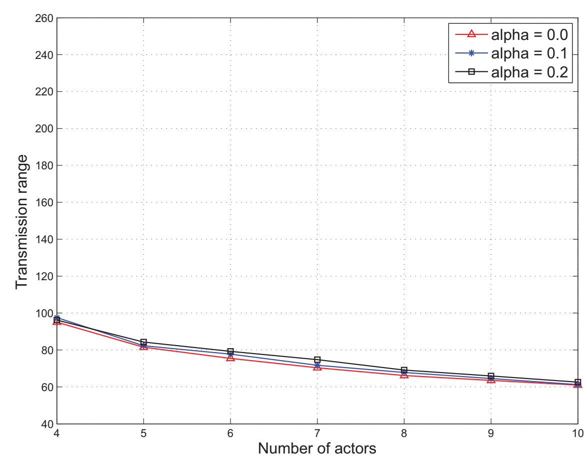

Fig. 15. Transmission range with the number of sensor nodes = 100 by different α(alpha) using *single-heuristic* with using hop bound = 3

VII. Summary and Concluding Remarks

In this paper, we studied the problem of the simultaneous optimization of transmission range and actor movement for multi-hop communication in WSANs. Even though an actor may be placed at any point in the field, we have presented a new and finite set of potential positions for locating actors that guarantee that the optimal solution is found within this set. We proposed a heuristic for the problem and compared the heuristic with an optimal ILP formulation. From our performance evaluation, we conclude that a significant decrease in total actor movement can be achieved at the cost of a small, or even negligible, increase in transmission range.

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