Optimal Transmission Range for Multi-hop Communication in Wireless Sensor and Actor Networks

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***Abstract*—Wireless sensor and actor networks (WSANs) consist of fixed sensor nodes and mobile actor nodes. Data is generated at the sensor nodes, and collected at the more powerful actor nodes. We consider the problem of finding the location for K actor nodes such that every sensor node is within d-hop from an actor node, where K and d are parameters of the problem. Our approach distinguishes itself in obtaining the minimum transmission radius necessary for such coverage to be possible. We provide an exact solution by via an integer-linear-programming formulation (ILP), and evaluate two heuristic approaches.**

I. INTRODUCTION

Wireless sensor networks (WSNs) have attracted the attention of many researchers due to the complexity in their network design and operation. A WSN is a collection of wireless sensor nodes that have constrained resources, such as battery power, and are deployed in a region of interest. In general, it is a data gathering network where sensor nodes are static and responsible for sampling their surroundings and reporting their data to predefined sink node [1].

As technology advances, WSNs has evolved into more complex systems. Originally, many papers considered a single sink node whose location is fixed. However, a mobile sink is useful in many applications, and can improve the performance of the network. Examples of applications benefiting from a mobile sink include battlefield monitoring and the prevention of wild fires [2]. Recently, improved hardware technology allows wireless sensor and actor networks (WSANs), which have attracted much interest. These consist of a set of wireless sensor nodes and a set of movable *actors*. Actors are powerful devices (e.g., unmanned vehicles, mobile robots), which may have the ability to change their location; although more powerful than sensor nodes, the energy in actors is still limited.

When a sensor node gathers data for specific event, it sends the information to an actor. Actors make decisions for various issues and perform appropriate actions based on the received information from sensor nodes and from other actors [3]. Moreover, multiple mobile actors improves network performance by increasing network lifetime and reducing data latency.

In WSANs, one major challenge is choosing the location of actors to achieve various network goals, such as maximizing sensor coverage, minimizing data collection delay, or balancing the load among actors. In general, each sensor node sends its data to other sensor nodes or actors that are located within its maximum transmission range.

In general, there are two cases to model for sensor communication, single-hop and multi-hop. In single-hop communication, each sensor node is within communication range of at least one actor. In multi-hop communication, data from some sensor nodes may have to traverse several other sensor nodes before reaching an actor node; this is usually due to a smaller transmission radius of the sensor nodes. Because the resources at sensor nodes are limited, and a longer transmission range implies more energy consumption, it is desirable to use multi-hop communication in WSANs.

Also, although all sensor nodes initially have same resource or battery, some nodes may consume more energy because an amount of communication is different among nodes because of their proximity to a point of interest. In the worse case, the network topology is reconstructed due to losing connectivity for sensor nodes with weak transmission power.

The number of hops between a sensor node and an actor also play an important role due to data latency [4]. That is, information from sensor nodes can be sent to actors within a given time constraint by being aware of the fact that latency of data is often proportional to the number of hops [5].

We are interested in minimizing the transmission range, but not at the expense of a significant increase in hop count. Thus, we focus instead on fixing an upper bound d on the number of hops that a sensor node may need to reach an actor, and finding the smallest transmission range r_{min} that satisfies this constraint.

We define the problem more formally as follows. We are given a set S of n sensors, $s_1, \ldots s_n$, which are randomly deployed in a two-dimensional plane. A total of K actors, $t_1, \ldots t_n$ are to be placed on the field. An *actor placement* $\mathcal{F}$ is a function that defines the location of each actor. An upper bound d is given on the allowed hop count from any sensor node to its closest actor.

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Let $min\text{-}hops(S, \mathcal{F}, r)$ be the minimum number of hops from any sensor node to any actor assuming each actor and sensor have a transmission range of r. Let $min\text{-}radius(S, \mathcal{F})$ be the smallest value of r such that $min\text{-}hops(S, \mathcal{F}, r) \le d$. Then, $\mathcal{F}$ is said to be a *solution* iff, for any actor placement $\mathcal{F}'$,

$$min\text{-}radius(S, \mathcal{F}) \le min\text{-}radius(S, \mathcal{F}')$$

II. Minimizing Single-Hop Transmission Range

Although we are interested in the multi-hop problem, we first discuss existing work on the single-hop version of the problem, and then return to the multi-hop version. Thus, consider finding the minimum radius r_{min} such that all sensor nodes are within a distance r_{min} from at least one of K actors (i.e., $d = 1$). This single-hop problem is equivalent to the euclidian p-center problem [9][10][11], whose solution may be obtained as follows.

Note that the problem is non-trivial because there are an infinite number of locations where actors may be placed (any point on the plane), and there are an infinite number of radii r to consider, since we do not assume r is discrete. At first glance, it appears to be a daunting task. However, although NP-hard, the problem is NP-complete, and a solution can be found by carefully selecting a finite set of possible actor locations and a finite set of radii. It can be shown [8][10] that the optimum radius r_{min} must belong to a finite set $R(S)$, where $|R(S)| \in \mathrm{O}(n^3)$. Also, for any r, if all sensor nodes can be covered by the actors using radius r, then the same can be accomplished if actor locations are chosen from some finite set $P(S, r)$, where $|P(S, r)| \in \mathrm{O}(n^2)$. I.e., if there is a solution with radius r, then there is also a solution with actor locations chosen from $P(S, r)$.

To solve the problem, assume there exists a procedure, $solve(S, K, r)$, to determine if, for a given radius r, it is possible to cover all sensor nodes with K actors. Obviously, if $r \ge r'$ and $solve(S, K, r')$ is successful, then so will $solve(S, K, r)$. Hence, a binary search is performed over the elements of $R(S)$ to find the smallest r satisfying $solve(S, K, r)$. This yields the optimum radius r_{min}.

The complexity of the problem arises not from the binary search ($O(\log n)$ steps), but from performing procedure $solve(S, K, r)$. As mentioned above, for a given radius r, a solution must exist by selecting actor positions from set $P(S, r)$, which is finite. Thus, $solve(S, K, r)$ may be implemented by testing all subsets of $P(S, r)$ of cardinality K, which has exponential complexity.

Due to space restrictions, we do not discuss why $P(S, r) \in \mathrm{O}(n^2)$ (See [6][9] for details). Briefly, however, we discuss why $R(S) \in \mathrm{O}(n^3)$.

Consider a subset S' of sensor nodes, and consider the *smallest* circle that covers each node in S'. In Fig. 1, we consider all possible cases. Fig. 1. (a) shows when the edge of the circle touches three or more sensors (drawn larger for clarity). Note that any three points in the plane define a unique circle that touches these three points. Fig.1. (b) shows when the smallest circle touches two nodes at opposite ends of its diameter. Fig. 1. (c) shows the degenerate case where $|S'| = 1$ and the radius is zero. Let $R(S)$ be the circles defined by all triples, doubles, and singletons that can be obtained from the set S of sensor nodes. Note that $|R(S)| \in \mathrm{O}(n^3)$. Thus, there are $\mathrm{O}(n^3)$ minimum circles (and their corresponding radii) that cover subsets of S. Also, note that any solution (i.e., with radius r_{min}) must contain at least one actor whose sensors are at the edge of its range, otherwise, the transmission range could be diminished. Hence, $r_{min} \in R(S)$.

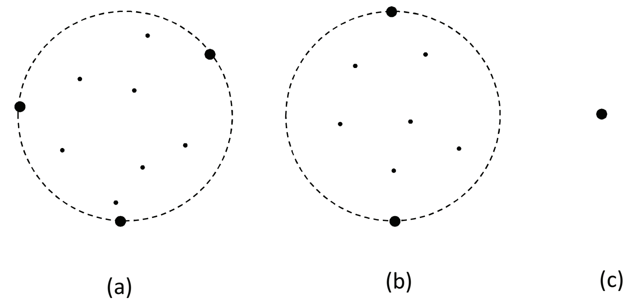

Fig. 1. Minimum circles covering a set of points.

III. Minimizing Multi-Hop Transmission Range

We next consider the multi-hop problem, i.e., when $d > 1$. We argue that an approach similar to the single-hop problem is applicable, as follows.

Consider a solution $\mathcal{F}$, when $d > 1$. Let t be an actor, and S_t the subset of sensor nodes that are covered in $\mathcal{F}$ by t in one-hop. From the earlier definition of $R(S)$, the radius necessary for t to cover S_t must be already included in $R(S)$. However, some sensor nodes will not communicate directly with an actor, and hence, the transmission range necessary to reach their next hop sensor may not be in our earlier definition of $R(S)$. Thus, we add to $R(S)$ the distance between every pair of sensor nodes (a total of $\mathrm{O}(n^2)$ values), to cover all possible next-hop choices for each sensor node. Note that $R(S)$ remains $\mathrm{O}(n^3)$, and its increase in size will only affect the binary search, so no significant complexity increase occurs.

It can be shown that the canonical locations in $P(S, r)$ will also generate a solution when $d > 1$. Due to space restrictions, this is shown in [7]. Thus, we can implement $solve(S, d, K, r)$ by exploring all subsets of size K from $P(S, r)$ and check if all sensors can reach an actor within d-hop.

Below, we present an ILP formulation for $solve(S, d, K, r)$, and two heuristic approximations.

IV. ILP Formulation for $solve(S, d, K, r)$

The following notation is used in our ILP formulation.

- P: set of potential locations for the actors.
- m: number of potential locations for actors, $m = |P|$.
- i: index for a sensor node, $1 \le i \le n$.
- h: index for possible next-hop positions,
  $1 \le h \le m$ for potential actor locations,
  $m + 1 \le h \le m + n$ for sensor nodes.

- p: index for a potential position in the set P,
  $1 \leq p \leq m$.
- d: given hop bound.
- r: maximum transmission range of sensor nodes.
- $\delta_{i,h}$: euclidean distance from sensor node i to possible next-hop h.
- U_h: integer variable indicating the order of nodes in the path,
  $U_1 = ... = U_m = 0$ and $1 \leq U_{m+1}, ..., U_{m+n} \leq d$.

Also, we define the following binary variables.

$$X_{i,h} = \begin{cases} 1, & \text{if if there is next hop from } i \text{ to } h \\ 0, & \text{otherwise.} \end{cases}$$

$$Z_p = \begin{cases} 1, & \text{if potential position } p \text{ is chosen as} \\ & \text{one of the } K \text{ positions} \\ 0, & \text{otherwise.} \end{cases}$$

Our objective function is to minimize total distance between sensor nodes and actors on condition that each sensor node can communicate with at least one actor within at most d-hop. Hence, we minimize the following value

$$\sum_{i=1}^{n} \sum_{h=1}^{m+n} \delta_{i,h} \cdot X_{i,h}$$

subject to the following constraints.

$$\sum_{p=1}^{m} Z_p = K$$

$$\sum_{h=1}^{m+n} X_{i,h} = 1, (\forall i, i \neq h)$$

$$\delta_{i,h} \cdot X_{i,h} \leq r, (\forall i, \forall h)$$

$$X_{i,p} \leq Z_p, (\forall i, \forall p)$$

$$U_h - U_i + d \cdot X_{i,h} \leq d - 1, (\forall i, \forall h)$$

V. Heuristic for $solve(S, d, K, r)$

In this section, we present two heuristics to approximate the minimum transmission radius that covers all sensors within d-hop. We simply refer to them as *greedy-1* and *greedy-2*. We borrow *greedy-1* from [10][4]. We then introduce our *greedy-2* heuristic, and their relative performance is evaluated in Section VII.

The basics of *greedy-1* are as follows. A total of K iterations, one per actor, are performed. The first actor position is chosen randomly from $P(S, r)$, and all sensor nodes within d hops from the chosen location are removed from the graph. At every iteration step, a new position is chosen from $P(S, r)$ that is the farthest away from all previously chosen positions. The sensor nodes within d-hop are then removed from the graph. The heuristic accepts r if all sensor nodes are removed from the graph after K iterations. The detailed steps are presented below.

Algorithm 1 (S, d, K, r)

$L \leftarrow \emptyset$
$T \leftarrow S$
for $p = 1$ to K **do**
  if $p = 1$ **then**
    set L_p to a random element from $P(S, r)$.
  else
    set L_p to the member of $P(S, r)$ such that L_p is the farthest one away from elements of L
  end if
  $L \leftarrow L \cup \{L_p\}$
  $T_d \leftarrow$ subset of T within d-hop of L_p
  $T \leftarrow T - T_d$
end for

Our *greedy-2* algorithm also performs K iterations, one per actor. However, it chooses actor positions based on the number of sensors that can be reached in one hop of radius r from said position. I.e., at each step, we choose the actor position that would maximize the number of neighbors of the actor. The sensor nodes within d-hop are then removed from the graph. The heuristic also accepts r if all sensor nodes are removed from the graph after K iterations. Its detailed steps are presented below.

Algorithm 2 (S, d, K, r)

$L \leftarrow \emptyset$
$T \leftarrow S$
for $p = 1$ to K **do**
  choose L_p from $P(S, r)$ such that L_p has the most neighbors from T within range r
  $L \leftarrow L \cup \{L_p\}$
  $T_d \leftarrow$ subset of T within d-hop of L_p
  $T \leftarrow T - T_d$
end for

VI. Approximation to the Optimum Radius

Above, we discussed how to obtain a finite number of radii that can be used to determine the minimum radius to connect each sensor node to some actor node within d-hop, provided there is a heuristic that finds the locations of the actors when given a fixed radius r.

Finding these actor locations is NP-hard [12]. However, of the two heuristics discussed earlier (Algorithm 1 and 2), it was shown in [4] that Algorithm 1 is within a factor of two from the optimal. That is, Algorithm 1 is guaranteed to place the actors such that the maximum number of hops from a sensor to an actor is no more than twice the optimal. Because of this, by combining the binary search over the radii with the actor placement heuristic in Algorithm 1, we are guaranteed to obtain a minimum radius r_{min} that is no more than twice the optimal radius r_{min}, simply as follows.

Assume that r_{min} is given as input to Algorithm 1. By definition of r_{min}, there is a placement of actors such that

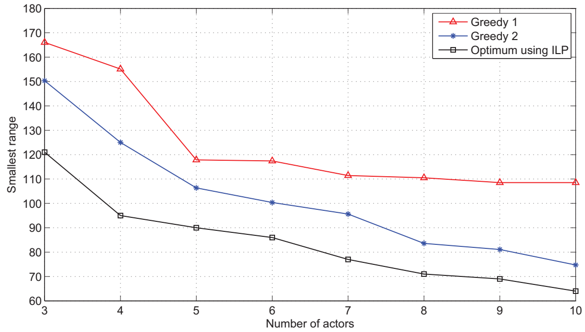

Fig. 2. Comparison of the smallest node transmission range with the number of nodes = 50 and hop bound = 2 by *greedy-1*, *greedy-2* and ILP

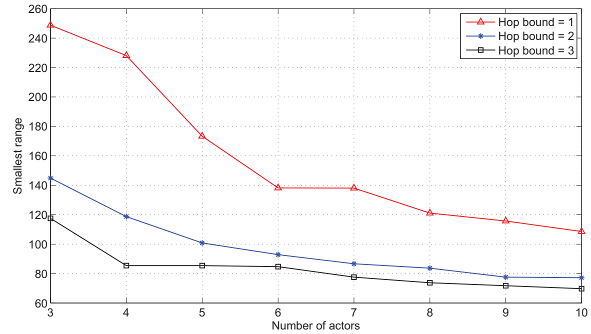

Fig. 3. Smallest node transmission range by different hop bound with the number of nodes = 100 using *greedy-2*

every sensor node reaches an actor in d-hop. Thus, from [4], Algorithm 1 will return a placement of actors such that every sensor reaches an actor within $(2 \cdot d)$-hop. Hence, the same actor placement can reach all sensors within d-hop if we use a radius r, where $r \geq 2 \cdot r_{min}$ (by skipping alternating nodes along the path). Therefore, when a radius r successfully completes a step of the binary search, we are guaranteed to obtain an actor placement satisfying the d-hop constraint, and we thus radius r is no more than twice the optimal.

VII. Experimental Evaluation

In this section, we simulated our experiments in a square-shaped sensor area of size 500 $\times$ 500 m^2. Initially, all sensor nodes are randomly deployed in the sensing field. We implemented our simulations with numbers of sensor nodes 50 and 100 respectively. Furthermore, the number of actors ranges between 3 and 10 and we considered 1-hop, 2-hop, 3-hop as hop bound d.

In the first simulation, we implemented the *greedy-1*, *greedy-2* and ILP. The comparison is shown in Fig. 2. If the number of actors increases, the node transmission radius decreases as a whole for *greedy-1*, *greedy-2* and ILP. Moreover, we can check that *greedy-2* significantly shows better performance than *greedy-1*. That is, *greedy-2* is closer than *greedy-1* to optimum based on ILP. For instance, for K = 7, although the transmission radius by *greedy-1* is about 111, *greedy-2* has about 95 for transmission range. By this simulation, we verify the theorem that *greedy-1* is never more than twice the optimum, as predicted in Section VI.

In the second scenario, we have run our *greedy-2* approach with 1-hop, 2-hop and 3-hop as d-hop bound. We can check the result for the number of sensor nodes is 100 in Fig. 3. When the number of actors increases, the transmission range decreases as the first simulation. Also, if the network has bigger d-hop bound, it has smaller node transmission range though the bigger d-hops bound increases data latency. It follows that there is a trade-off between node transmission range and hop bound.

VIII. Concluding Remarks

In this paper, we proposed a heuristic approach whose goal is to find the smallest node transmission range for K actors to cover a collection of sensor nodes within d-hop in WSANs. Moreover, the minimum transmission range r_{min} is guaranteed to be found using the ILP formulation presented above. As future work, we consider a heterogeneous environment where the transmission range for each sensor can be different. We will also investigate whether the heuristic in Algorithm 2, which behaves on average better than Algorithm 1, is able to guarantee a solution within a constant of the optimum.

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