PHY 444 Final Exam **Name** _____

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate.
- If you need more space, you may use the back of a page or the extra sheet on the last page and write *On back of page #* in the problem. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods. **Pace yourself!**

If you are stuck, or running out of time, indicate as completely as possible, the physics and steps you would take to tackle the problem. Also, indicate any relevant information that you would use.

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	**22**	
2	**18**	
3	**14**	
4	**14**	
5	**16**	
6	**11**	
7	**15**	
Total	**110**	

$R_H = 1.097\times10^{7}\,\mathrm{m}^{-1}$, $h = 6.626\times10^{-34}\,\mathrm{Js}$, $\hbar = 1.055\times10^{-34}\,\mathrm{Js}$, $1\mathrm{eV} = 1.6\times10^{-19}\,\mathrm{J}$,
$m_e = 9.11\times10^{-31}\mathrm{kg}$, $m_p = 1.67\times10^{-27}\mathrm{kg}$, $a_0 = 0.5292\times10^{-10}\,m$

Spin-1 Matrices: $\hat{S}_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix}$, $\hat{S}_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix}0&-i&0\\i&0&-i\\0&i&0\end{pmatrix}$, $\hat{S}_z = \hbar\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}$.

Eigenvectors of S_x: $\frac{1}{2}\begin{pmatrix}1\\\pm\sqrt{2}\\1\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\end{pmatrix}$. Eigenvectors of S_y: $\frac{1}{2}\begin{pmatrix}\mp1\\\sqrt{2}i\\\pm1\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\end{pmatrix}$.

Have a good break!

PHY 444 Final Exam **Name** _____

1. (6 pts) **Two Point Short Answers:** Answer the following
   a. Give an equation relating $\langle p|\psi\rangle$ to $\langle x|\psi\rangle$.

   b. Write $\hat{S}_y$ in terms of $\hat{S}_+$ and $\hat{S}_-$.

   c. What is hyperfine structure?

2. (4 pts) A spin-½ particle with charge $-e$ and mass m is placed in a uniform magnetic field $\mathbf{B} = B_0\mathbf{i}$ in a state $|\psi(0)\rangle = |+z\rangle$.
   a. Write the Hamiltonian, $\hat{H} = \omega_0 \hat{S}_x$, in terms of m, e, B_0 and a spin operator.

   b. Describe the time evolution of the particle.

3. (12 pts) **Three Point Short Answers:** Answer the following
   a. A hydrogen electron absorbs a photon and its state changes from $|\psi_{200}\rangle$ to $|\psi_{521}\rangle$. What is the frequency of the photon?

   b. List the known quantum numbers for the electronic configuration $2d_{\frac{3}{2}}$.

   c. Name two types of relativistic perturbations of the hydrogen atom.

   d. A spin up hydrogen electron is in the state $|4,2,1\rangle$. Measurements of $\hat{L}_z$, $\hat{L}^2$ and $\hat{J}_z$ yield what three values?

4. (6 pts) Evaluate the following
   a. $[\hat{J}_{-}\hat{J}_{+},\hat{J}_{x}]^{\dagger} =$

   b. $\hat{S}_{-}\left|\frac{3}{2},-\frac{1}{2}\right\rangle =$

   c. $_{y}\left\langle\frac{1}{2},-\frac{1}{2}\middle|\frac{1}{2},\frac{1}{2}\right\rangle_{z} =$

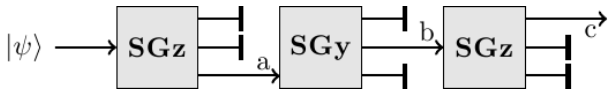
5. (8 pts) $\psi(\theta,\varphi)=C\left(5Y_{43}+Y_{63}+3Y_{60}\right)$ is the angular part of a hydrogen wavefunction, where $Y_{\ell m}=Y_{\ell m}(\theta,\varphi)$ are normalized spherical harmonics.
   a. What is the normalization constant, C?

   b. What is the probability of finding an atom in the state with $m=3$?

   c. What are the expectation values of $\hat{L}^2$ and $\hat{L}_z$?

6. (4 pts) Consider the potential $V(x)=\begin{cases}-\alpha\dfrac{\hbar^2}{2m}\delta(x), & |x|<\dfrac{a}{2}\\ \infty, & |x|>\dfrac{a}{2}\end{cases}$ for $\alpha>0$.
   Derive the transcendental equation for the bound state energy, $E<0$.

7. (4 pts) A beam of spin-1 particles enters the sequence of Stern-Gerlach devices as depicted below with two of the three channels block after each transition. Find the probability that particles follow the given channel by first writing the appropriate probability amplitude indicating the basis needed. (For example: ${}_z\langle 1,1|1,1\rangle_x$.)



   a. Probability that particles pass from a to b: _____

   b. Probability that particles pass from b to c: _____

8. (10 pts) For the harmonic oscillator Hamiltonian, $\hat{H} = \left(\hat{a}^\dagger \hat{a} + \frac{1}{2}\right)\hbar\omega$ with $\hat{x} = \beta(\hat{a} + \hat{a}^\dagger)$, let $|\psi(0)\rangle = \frac{1}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle$ and do the following:

   a. Find $\langle E \rangle$.

   b. Compute $\langle x \rangle$ in terms of β.

   c. Find $|\psi(t)\rangle = Ne^{i\delta_1}\left[|0\rangle + e^{i\delta_2}|1\rangle\right]$ with explicit constants determined.

   d. Determine $\langle x(t)\rangle = \langle \psi(t)|\hat{x}|\psi(t)\rangle$ and simplify.

9. (4 pts) Find the correctly normalized state $Y_{1,0}(\theta,\phi)$ using the raising operator, $\hat{L}_+ = \frac{\hbar}{i} e^{i\phi}\left(i\frac{\partial}{\partial\theta} - \cot\theta\frac{\partial}{\partial\phi}\right)$ and the state $Y_{1,-1}(\theta,\phi) = \sqrt{\frac{3}{8\pi}}\sin\theta e^{-i\phi}$.

10. (10 pts) Consider the state vector $|\psi\rangle = N\left(|+z\rangle + 3i|-z\rangle\right)$.

a. Normalize this state vector.

b. What is the probability that a beam of particles in this state

i. Entering an SGz device will measure $S_z = -\frac{\hbar}{2}$, or

ii. Entering an SGy device will measure $S_y = \frac{\hbar}{2}$?

c. Calculate $\langle S_z \rangle$.

d. Find the uncertainty, ΔS_z.

11. (6 pts) Let $\psi(x) = Ae^{-b^2x^2/2}$ be the ground state of a one-dimensional harmonic oscillator.

    a. Assuming A is real, normalize the wavefunction.

    b. What is the potential and energy in terms of $\hbar$, m, and b?

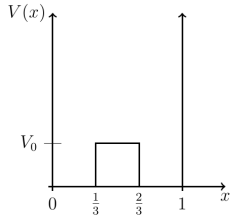
    c. What is the average potential energy?

12. (10 pts) Consider a particle in the infinite square well $V(x) = \begin{cases} 0, & 0 \le x \le 1, \\ \infty, & \text{elsewhere.} \end{cases}$

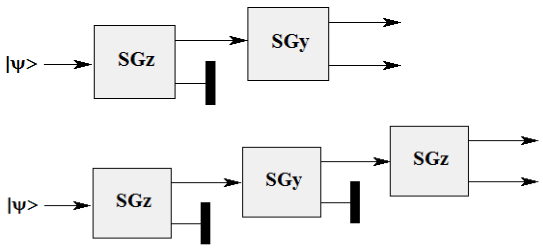
    a. What are $\psi_4(x)$ and E_4?

    b. If the particle is in state $\psi_4(x)$, what is the probability of finding the particle in the interval $\frac{1}{4} \le x \le \frac{1}{2}$?

    c. Consider the following perturbation to the potential well with $V_0 = \frac{\pi^2\hbar^2}{2m}$ added in $\frac{1}{3} \le x \le \frac{2}{3}$. What is the first order energy shift for E_4?



13. (2 pts) A spin-$\frac{1}{2}$ particle in state $|\psi\rangle = \frac{3e^{i\pi/4}}{\sqrt{13}}|+z\rangle + \frac{2}{\sqrt{13}}|-z\rangle$ enters each device. Record the probability that the end device will measure $S = -\frac{\hbar}{2}$.



_____

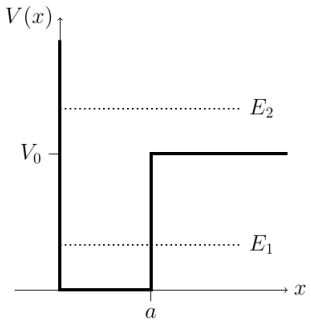
_____

14. (9 pts) Consider the semi-infinite square well potential below.

a. For energies $0 < E_1 < V_0$,

i. Write the general forms for solutions for $0 < x < a$ and $x > a$.

ii. Show that the energy must satisfy: $\cot\frac{a\sqrt{2mE_1}}{\hbar} = -\sqrt{\frac{V_0 - E_1}{E_1}}$.



iii. Describe how the bound state energies are determined.

b. A particle with energy $E_2 > V_0$ is incident from the right. Describe the state of the particle in the regions $0 < x < a$ and $x > a$.

15. (3 pts) The hydrogen ground state probability for finding the electron between r and $r+dr$ is $dP=\frac{4}{a_0^3}r^2e^{-2r/a_0}dr$, where a_0 is the Bohr radius. Determine the average position, $\langle r\rangle$.

16. (5 pts) Consider the Hamiltonian $H=\begin{pmatrix}1 & \varepsilon & 0\\ \varepsilon & 2 & 0\\ 0 & 0 & 1\end{pmatrix}\hbar\omega$.

a. What are the energies and eigenstates of the unperturbed system ($\varepsilon=0$)?

b. Determine the first order energy shifts.

17. (7 pts) Let $\psi(x)=\langle x|\psi\rangle=\begin{cases}Ne^{-ax/2}, & x>0,\\ 0, & x<0,\end{cases}$ where $a>0$.

a. Find $\langle x\rangle,\langle x^2\rangle$. [Hint: Tap your inner Feynman.]

b. What is Δx?

Bonus: Find the momentum wavefunction for Problem 17.

Extra Space