

MAT 463/563 Review Topics I

I. First Order Differential Equations

a. Separation of Variables

$$i. \frac{dy}{dt} = f(t)g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(t) dt$$

ii. General Solutions – Implicit and Explicit

iii. Initial Value Problems – Particular Solutions

b. Linear Differential Equations

i. Find integrating factors and solve initial value problems

$$ii. y' + a(x)y = f(x)$$

$$\mu(x) = \exp \int a(x) dx \Rightarrow (\mu y)' = \mu f$$

$$y(x) = \frac{1}{\mu(x)} \left[\int \mu(t)f(t) dt + C \right]$$

II. Second Order Differential Equations

a. Homogeneous, Constant Coefficient Equations $y'' + ay' + by = 0$

b. Solutions - $y(x) = e^{rx}$, $r^2 + ar + b = 0$.

i. Two, real distinct solutions $y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$

ii. One real solution $y = (c_1 + c_2 x)e^{rx}$

iii. Two complex conjugate solutions $y = (c_1 \cos \beta x + c_2 \sin \beta x)e^{\alpha x}$

c. $ax^2 y'' + bxy' + cy = 0$, $y = y(x)$

i. Solve characteristic equation $ar(r-1) + br + c = 0$.

ii. Three Cases:

1. $k_1 x^{r_1} + k_2 x^{r_2}$

2. $x^r (k_1 + k_2 \ln |x|)$

3. $x^\alpha [k_1 \cos(\beta \ln |x|) + k_2 \sin(\beta \ln |x|)]$, $r = \alpha \pm i\beta$.

III. Linear Systems of Differential Equations

a. Phase Plane and Phase Portraits

i. Find solutions from phase portrait.

ii. Equilibrium Solutions

iii. y vs x and $x(t)$, $y(t)$ plots

b. Eigenvalue Problems – Solve for eigenvalues and eigenfunctions.

c. Solution of systems – Use eigenvalues and eigenfunctions to construct solutions to systems.

d. Understand classification of Equilibrium Points and Connection to Phase Portraits, Eigenvalues and Solution Behavior.

e. Types: Stable/Unstable, Nodes, Foci, Centers, Degenerate Nodes, and Saddles.

f. Fundamental Matrix, Principal Matrix

- IV. Methods of Integration
- Substitution
 - Integration by parts $\int u dv = uv - \int v du$
 - Trigonometric Integrals $\int \sin^n x dx, \int \cos^n x dx,$
 - Trigonometric Substitution
 - $x^2 + a^2$ - tangent substitution
 - $a^2 - x^2$ - sine substitution
 - Integration using Partial Fraction Decomposition
- V. Integrals you should be able to do (or similar ones)

$\int x^n dx$	$\int \frac{1}{x} dx$	$\int e^{ax} dx$	$\int a^x dx$
$\int \sin ax dx$	$\int \cos ax dx$	$\int \sec^2 ax dx$	$\int \csc^2 ax dx$
$\int \sec x \tan x dx$	$\int \csc x \cot x dx$	$\int \sinh ax dx$	$\int \cosh ax dx$
$\int \tan ax dx$	$\int \cot ax dx$	$\int \frac{1}{x^2 + a^2} dx$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx$
$\int \sec ax dx$	$\int \ln x dx$	$\int x^n e^{ax} dx$	$\int \frac{1}{x^2 - a^2} dx$
$\int \sin^2 ax dx$	$\int \cos^2 ax dx$	$\int \sin ax \cos bxdx$	$\int \sin ax \sin bxdx$
$\int \frac{dx}{a + bx}$	$\int \frac{dx}{(x - a)(x - b)}$	$\int e^{ax} \cos bxdx$	$\int e^{ax} \sin bxdx$