MAT 463/563 Exam I **Name** _____

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate..
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space or the attached blank sheet. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use. Do not spend too much time on one problem. **Pace yourself**

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	**13**	
2	**10**	
3	**10**	
4	**17**	
Total	**50**	

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1. (13 pts) Solve the following equations:
   a. $y' = x(y^2 + 1),\ y(0) = 1.$

   b. $y' - (\tan x)y = x.$

   c. $x^2 y'' + xy' - y = 0.$

   d. $x'' + 3x' - 18x = 3e^{2t}$

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2. (5 pts) Consider the following system $\begin{aligned} x' &= x - 2y \\ y' &= 2x + y \end{aligned}$.

   a. Classify the equilibrium point of the system.

   b. Rewrite the system as a second order differential equation in $x(t)$.

3. (5 pts) For the system $\begin{aligned} x' &= 2y \\ y' &= 3x \end{aligned}$

   a. Find an expression for the slope of the trajectories, $\frac{dy}{dx}$.

   b. Solve the equation found in Part a. and describe the family of solutions you would see in the phase plane.

   c. What type of equilibrium point is this?

4. (10 pts) Consider the system $\begin{aligned} x' &= 5x + 3y \\ y' &= -6x - 4y \end{aligned}$.

   a. Find the eigenvalues of the coefficient matrix.

   b. Find the eigenvectors of the coefficient matrix.

   c. Construct the Fundamental matrix.

   d. Write the general solution for *x(t)* and *y(t)*.

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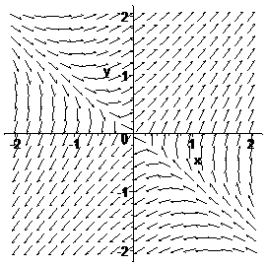
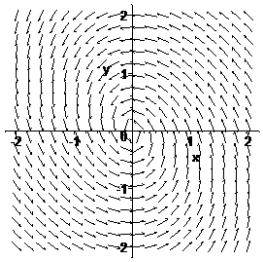
5. (10 pts) In the following figures classify the equilibrium point. For each equilibrium point you find, describe the form of the eigenvalues (real, complex, positive real part, negative, etc) that you would expect.


A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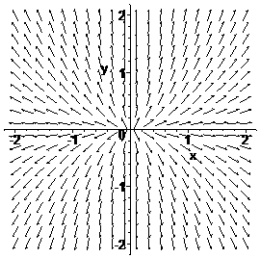
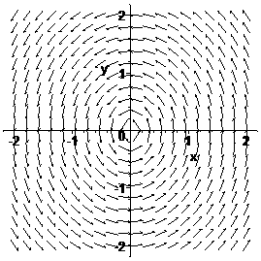

C


D

6. (4 pts) Consider the system $\begin{aligned} x' &= x - y \\ y' &= x + y \end{aligned}$.

   a. For $r = \sqrt{x^2 + y^2}$, write $\dfrac{dr}{dt}$ in polar form for this system.

   b. What does the solution of the equation in Part a. tell you about the behavior of the equilibrium point in this system?

7. (3 pts) Let the coefficient matrix of two first order linear differential equations have only one real eigenvalue. One solution is given by $\mathbf{x} = \mathbf{v}_1 e^{\lambda t}$. Describe how one determines a second linearly independent solution.

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Extra Space