MAT 463/563 Exam II   Name ________________________

Instructions:

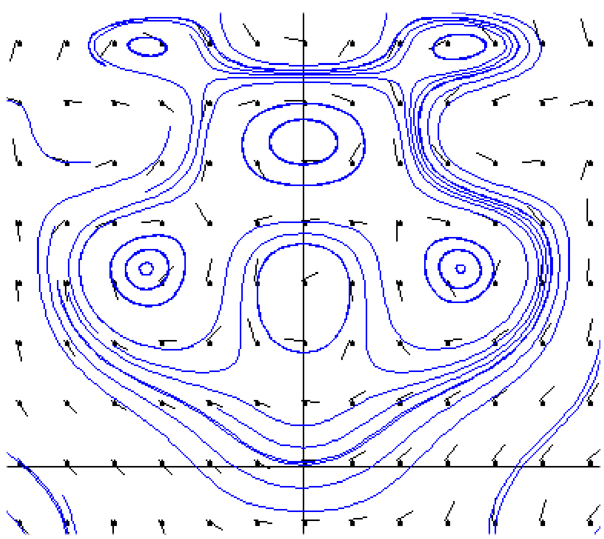
- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate..
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use. Do not spend too much time on one problem. **Pace yourself.**

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	12	
2	19	
3	14	
4	5	
Total	50	



What do you see?

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1. (8 pts) In the following problems locate and classify the equilibrium solutions of the system.

   a. $$\begin{aligned} x' &= y + x^3 \\ y' &= -x + y^3 \end{aligned}$$

   b. $$\begin{aligned} x' &= 2x - 3xy \\ y' &= y + xy \end{aligned}$$

2. (4 pts) Consider the following Lotka-Volterra population model where x and y are measured in hundreds of organisms. $$\begin{aligned} x' &= -x - 2x^2 + xy, \\ y' &= -y + 7xy - 2y^2. \end{aligned}$$

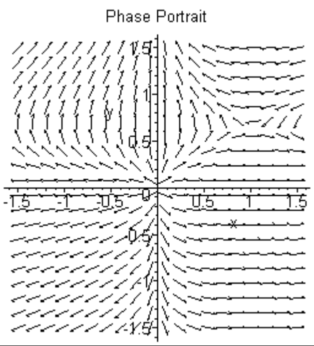
   a. Is this a predator prey, or competitive model?

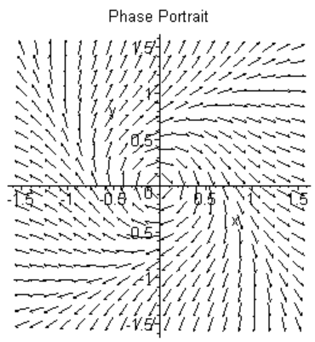
   b. For population y, what does the y^2 term represent?

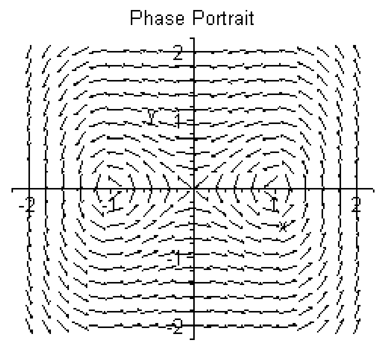
   c. What happens if both populations are small initially?

   d. **Bonus** Are there any initial conditions that would lead to continued existence of one, or both, populations without the fear of extinction? (Show work!)

3. (12 pts) In the following figures describe the types of behavior you see and sketch typical orbits supporting your observations. For each fixed point you find, describe the form of the eigenvalues (real, complex, positive, etc) of the linearized problem.







A	B	C

4. (7 pts) Consider the system $\begin{array}{l} x' = y \\ y' = x - x^3 - by \end{array}$

   a. For $b = 0$ find the equilibrium points and sketch the orbits you would expect in the square $[-1.5, 1.5] \times [-1.5 \times 1.5]$.

   b. Sketch how this figure would look if $b \neq 0$.

5. (6 pts) Consider the family of differential equations $\dfrac{dy}{dx} = x(\mu - x^2)$. Construct the bifurcation diagram. What kind of bifurcation is it? ____________________

6. (8 pts) Consider the polar system $\frac{dr}{dt} = r(1-r^2)(4-r^2)$, $\frac{d\theta}{dt} = -1$, $r \geq 0$.
   a. Draw a phase line for the *r* equation, labeling the equilibria, and their stability.

   b. Sketch the limit cycles for this system in the *xy*-plane.

   c. In the above figure (part b) sketch one orbit in each region being careful about choosing the correct direction for any spirals.

7. (5 pts) Consider the two tank problem: Pure water enters container A at a rate of 3.0 gal/min and the well stirred mixture flows into container B at 4.0 gal/min. The mixture in B leaks back into A at one gal/min and leaves the system at 3.0 gal/min. Assume each container holds 100 gallons, tank A initially has 10.0 pounds of salt and B has none. Set up the initial value problem for this system.

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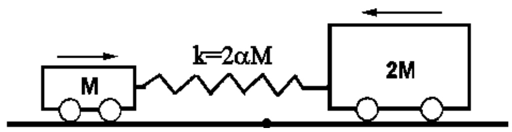
Extra Problems for Practice.

8. (5 pts) Two masses are attached by a spring with spring constant $k = 2\alpha M$, as shown below.



Derive the system of first order equations that models the motion of these masses in the form $\dot{\mathbf{x}} = A\mathbf{x}$.

9. Define the following:
   a. Hopf bifurcation.
   b. Nullcline.
   c. Bifurcation point.
   d. Hyperbolic point.
   e. Poincaré surface of section.
   f. Phase space

10. Consider $\frac{dx}{dt} = \mu - x - e^{-x}$. Show that as μ is varied, the system undergoes a saddle-node bifurcation. [Probably a grad student problem.]

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11. Consider the nonhomogeneous problem: $\mathbf{x}' = \begin{pmatrix} -5 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 6 \\ -1 \end{pmatrix} e^{2t}$.

   a. Construct the fundamental solution matrix.

   b. Find the principal solution matrix.

   c. Determine the particular solution for this problem.

12. In the stability diagram for planar systems carefully indicate the locations of: centers, stable spirals, stable nodes, saddles, degenerate sources, and unstable lines of equilibria.