

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate.
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space or the attached blank sheets. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use.

Pace yourself – do not spend more than 15 minutes per page on your first pass.

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	16	
2	17	
3	13	
4	14	
Total	60	

Bonus: Use the generating function $g(x,t) = \frac{1}{\sqrt{1-2xt+t^2}}$ to find $P_4(0)$.

Note: $(n+1)P_{n+1}(x) - (2n+1)xP_n(x) + nP_{n-1}(x) = 0$

1. (12 pts) Do the following:

a. Simplify: $\frac{2+i}{1-4i}$.

b. Convert to polar form: $i - \sqrt{3}$.

c. List all of the roots $(-i)^{1/3}$ in standard, exact form, $z = a + bi$, without any trigonometric functions.

d. Find all values of z such that $e^z = -3$.

2. (2 pts) State the Cauchy-Riemann equations,

3. (2 pts) Evaluate $\int_0^{\infty} e^{-3x^2/4} dx$.

4. (3 pts) Find the real and imaginary parts of $f(z) = \sin\left(z - \frac{\pi}{2}\right)$ in the simplest form.

5. (3 pts) Let $g(z_0) = \oint_C \frac{z^3 + 3z}{(z - z_0)} dz$. Find $g(1)$ for the following simple, closed,

positively oriented contours:

a. $|z - 1| = 1$;

b. $|z - 3i| = 2$.

6. (5 pts) Classify the singularities of $f(z) = \frac{z^2 - 4}{(z - 2)(z^2 + 4)^2}$ and find the residue at $z = 2i$.

7. (3 pts) Use the three term recursion relation to find $P_3(x)$.

[Recall, $P_2(x) = \frac{1}{2}(3x^2 - 1)$.]

8. (3 pts) Find the value of $\frac{\Gamma(7)}{7!!} \Gamma\left(\frac{5}{2}\right)$ in terms of $\sqrt{\pi}$.

9. (3 pts) Consider the function $u(x, y) = xy$. Find the harmonic conjugate $v(x, y)$.

10. (4 pts) Find the unique expansion of $f(z) = \frac{1}{(z+2)(z-3)}$ in powers of z that converges in the annulus $2 < |z| < 3$.

11. (3 pts) Compute $\int_C \frac{dz}{z^2}$ for C a positively oriented arc of radius 2 from $z = 2i$ to $z = -2$.

12. (3 pts) Use the three term recursion formula to evaluate $\int_{-1}^1 xP_2(x)P_3(x) dx$.

13. (4 pts) Find the Laurent series expansion for $f(z) = \frac{\sin z - z}{z^6}$ about $z = 0$. Classify the singularity at $z = 0$. _____

14. (6 pts) Do **two** of the following integrals:

a. $\oint_{|z|=3} \frac{z^2 + 1}{z^2(z-2)} dz$

b. $\int_0^{\infty} \frac{\cos x}{1+x^2} dx$

c. $\int_0^{2\pi} \frac{d\theta}{3+2\cos\theta}$

15. (4 pts) Given the basis $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$, use the Gram-Schmidt process to construct an orthogonal basis.

MAT 367 Exam II

Name _____

Extra Space