MAT 367 Final Exam **Name** ________________________

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate.
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space or the attached blank sheet. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use. Do not spend too much time on one problem. **Pace yourself.**

Pay attention to the point distribution. Not all problems have the same weight.

Page	**Pts**	**Score**
1	**16**	
2	**15**	
3	**17**	
4	**14**	
5	**12**	
6	**16**	
7	**10**	
Total	**100**	

Have a good summer!

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1. (6 pts) Finish the following statements:
   a. The Fourier transform of a Gaussian function is a ___________________.

   b. Define $Ш(x) =$ ______________________ .

   c. The convolution of two box functions on $(-\infty, \infty)$ is a

      ____.

   d. The function $f(t) = \begin{cases} t, & 0 \le t \le 1 \\ 0, & t>1, \end{cases}$ is given in terms of Heaviside

      functions as ____.

   e. To capture a frequency of 30 Hz, you need to use a sampling frequency of at least ______ Hz.

   f. If $f(z)$ is analytic on and inside $|z - z_0| = \rho$, then $\oint\limits_{|z-z_0|=\rho,} \frac{f(z)}{z - z_0} dz =$ _____.

2. (4 pts) Find the inverse Fourier transform, $f(x)$, of $\hat{f}(k) = \frac{1}{1+k^2}$.

3. (6 pts) Evaluate the following:

   a. $\int_0^5 \delta(x+1) \cos \pi x \, dx$

   b. $\int_{-\infty}^{\infty} e^{-|x|} \delta(x^2 + 2x - 3) \, dx$

4. (3 pts) Evaluate $\int_{-\infty}^{\infty} \cos 3x\, e^{ikx}\, dx$ in terms of delta functions by first writing the integrand in terms of exponentials.]

5. (9 pts) Consider the sequence of functions defined by $f_n(x) = \sqrt{n}e^{-nx^2}$, $|x| < \infty$.
   a. Sketch $f_n(x)$ and label the location and height of the peak.

   b. Determine the width, Δx, of f_n, [FWHM, full width at half maximum].

   c. Find the area under each function in the sequence.

   d. Based on these results, find $\lim_{n\to\infty} f_n(x) =$

6. (3 pts) Evaluate $\oint_{|z-1|=2} \frac{z^2+4}{z^2(z-4)}\, dz$

7. (3 pts) Evaluate $\int_{-\infty}^{\infty} e^{-x^2-2x}\,dx$.

8. (7 pts) Find the Laplace transforms and simplify:
   a. $L[e^{-2t}(\cos 5t - \sin 5t)] =$

   b. $L[t^2 \sinh t] =$

   c. $L\left[H\left(t-\frac{\pi}{2}\right)\sin t\right] =$

9. (7 pts) Find the inverse Laplace transforms:
   a. $L^{-1}\left[\frac{s+1}{s(s^2+3)}\right] =$

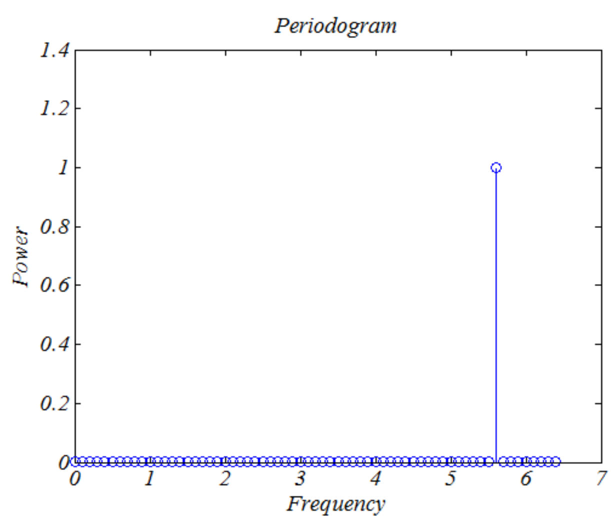
   b. $L^{-1}\left[\frac{s}{s^2+2s+5}e^{-2s}\right] =$

10. (6 pts) Consider the impulse train $f(t) = \sum_{n=-\infty}^{\infty} \delta\left(t - nT_s\right) = \text{comb}_{T_s}(t)$.

   a. Compute the Fourier coefficients $c_k = \frac{1}{T_s}\int_0^{T_s} f(t)e^{i\omega t}\,dt$ in the representation $f(t) = \sum_{n=-\infty}^{\infty} c_n e^{in\omega_s t}$, where $\omega_s = \frac{2\pi}{T_s}$.

   b. Use the last result to prove that the Fourier transform of the comb function can be written as $\hat{f}(\omega) = \omega_s \text{comb}_{\omega_s}(\omega)$.

11. (8 pts) Consider the spectra below. In each case the data was sampled at 128 points for $T = 10$ seconds. [Do not forget the units in your answers.]



   a. What is the sampling rate, f_s? ________
   b. What is Δf? ________
   c. Compute the maximum frequency in the plot? ________
   d. Determine the main frequency of the spike in the plot. ________
   e. Write a possible function giving rise to this signal:
      $y(t) =$ ____________________
   f. If the signal were sampled instead at $f_s = 10.0$ Hz, what frequency would appear? ________

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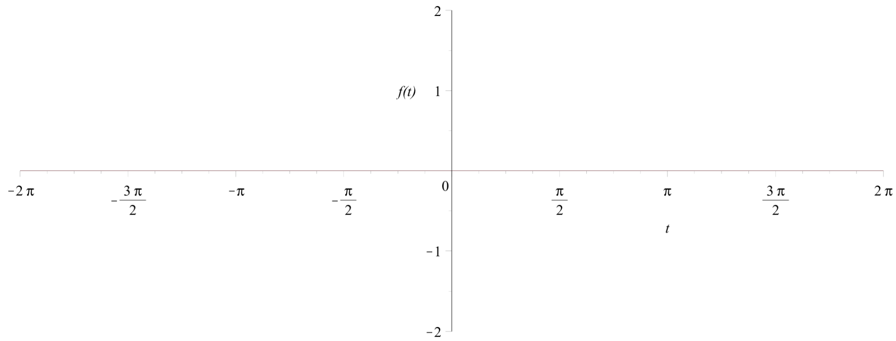
12. (12 pts) Consider the finite wave train $f(t)=\begin{cases} 2\cos 2t, & -\pi \le t \le \pi. \\ 0, & \text{otherwise.} \end{cases}$

a. Neatly plot this function on the axes below.

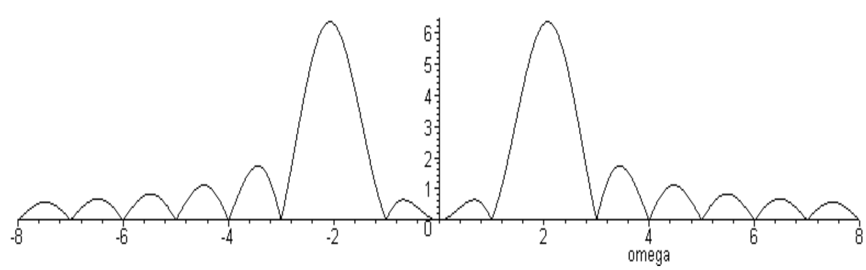


b. Find the Fourier transform, $\hat{f}(\omega)$, of $f(t)$ and simplify.

c. Find the Fourier coefficients in the Fourier series expansion on $[-\pi,\pi]$,

$f(t)=\dfrac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt.$ Be careful before doing too many integrals.

d. A plot of $\hat{f}(\omega)$ from part b is shown below. Explain how the Fourier coefficients in part c are related to the Fourier transform in part b.



13. (4 pts)) Find the value of $\sum_{n=0}^{9} \cos\frac{n\pi k}{5}$ for $k = 0,1,\ldots,10$ without a calculator.

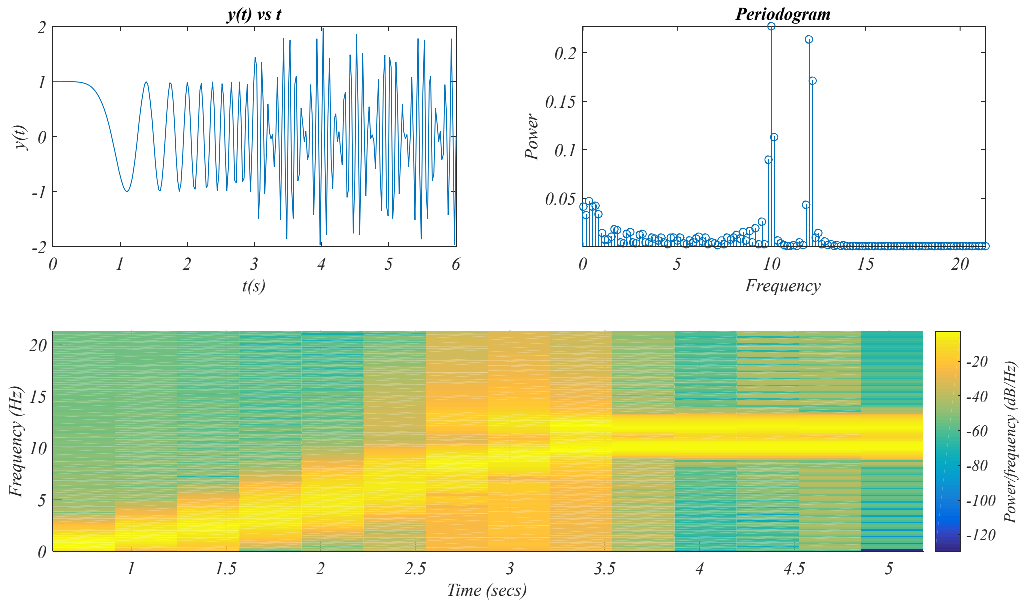
14. (8 pts) Let $F(s) = \frac{1}{(s-2)(s+3)}$.

    a. Use the Bromwich integral to find the inverse Laplace transform.

    b. Use the Convolution Theorem to find the inverse Laplace transform.

15. (4 pts) Use Laplace transforms to solve $y'' + 9y = 2\delta(t-4)$ subject to the initial conditions $y(0) = 1, y'(0) = 0$.

16. (4 pts) From the below plots what can you say about the signal? Be specific about the frequency values and types of signals present.



17. (2 pts) Evaluate and simplify $\Gamma\left(\frac{9}{2}\right) =$

18. (4 pts) Volterra (1860 – 1940) studied population models leading to integral equations such as $y(t) = \sin t - 2\int_0^t \cos(t-u)y(u)\,du$. Solve this Volterra integral equation for the unknown function, $y(t)$, by first taking the Laplace transform of the equation, solving for $Y(s)$, and taking the inverse transform.

Extra Space