MAT 365 Review Topics for Final

The following is a review of the topics covered since the second exam. However, this does not mean that you should not know material from the first half of the course. As you know, there are several things that you need to know how to do in order to do the second half of the course, such as standard vector operations; parametrizing lines, curves and planes; computing work and flow integrals; working with differential forms; doing line and multiple integrals; Jacobians and determinants; directional derivatives and gradients; and, sketching surfaces using level curves.

I. Integration by Pullback
   a. Transformations and Jacobians
   b. Surface Integrals – Parametric Surfaces

$$Areas(S) = \int_S ds = \int_{\mathbf{F}^{-1}(S)} \left| \frac{\partial \mathbf{F}}{\partial u} \times \frac{\partial \mathbf{F}}{\partial v} \right| dudv$$

$$\int_S f(x,y,z)\, ds = \int_{\mathbf{F}^{-1}(S)} f(x(u,v), y(u,v), z(u,v)) \left| \frac{\partial \mathbf{F}}{\partial u} \times \frac{\partial \mathbf{F}}{\partial v} \right| dudv$$

   c. Rate of Flow Across S: $\int_S v_1\, dydz + v_2\, dzdx + v_3\, dxdy = \int_R \mathbf{v} \bullet \mathbf{n}\, ds$

II. Techniques of Differential Calculus
   a. Implicit Differentiation (Implicit Function Theorem)
   b. Inverse Function Theorem
   c. Extrema – Stationary Points, Saddles, Hessian
   d. Second Order Taylor Formula for $F : \mathbf{R}^n \to \mathbf{R}$
   e. Classification of Extrema
   f. Lagrange Multipliers – with one constraint $g(x,y,z) = c,\ \nabla f = l \nabla g$

III. The Fundamental Theorem of Calculus
   a. $\int_{\partial M} w = \int_M dw$
   b. Green's Theorem: $\int_{\partial M} f\, dx + g\, dy = \int_M \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dxdy.$
   c. Gauss' Theorem:

$$\int_{\partial M} f\, dydz + g\, dzdx + h\, dxdy = \int_M \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z} \right) dxdydz.$$

$$\int_{\partial M} \mathbf{F} \bullet \mathbf{n}\, ds = \int_M \nabla \bullet \mathbf{F}\, dV.$$

   d. Stokes' Theorem:

$$\int_{\partial M} f\, dx + g\, dy + h\, dz = \int_M \left(\frac{\partial h}{\partial y} - \frac{\partial g}{\partial z} \right) dydz + \left(\frac{\partial f}{\partial z} - \frac{\partial h}{\partial x} \right) dzdx + \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dxdy.$$

$$\int_{\partial M} \mathbf{F} \bullet d\mathbf{r} = \int_M \nabla \times \mathbf{F} \bullet \mathbf{n}\, ds.$$

   e. Exact and Closed Forms
   f. Finding Antidifferentials
   g. Path Independence
   h. Poincare's Lemma
   i. Vector Operators: Gradient ∇, Divergence $\nabla \bullet$, Curl $\nabla \times$
   j. $\nabla \times (\nabla f) = 0, \nabla \bullet (\nabla \times \mathbf{F}) = 0$
   k. Irrotational $\nabla \times \mathbf{F} = 0 \Rightarrow \exists f \ .\ni.\ \mathbf{F} = \nabla f$,
      Incompresssible $\nabla \bullet \mathbf{F} = 0 \Rightarrow \exists \mathbf{G} \ .\ni.\ \mathbf{F} = \nabla \times \mathbf{G}$
   l. Irrotational $\Rightarrow \nabla \bullet (\nabla f) = 0$, or f is harmonic (i.e., it satisfies Laplace's Equation $\nabla^2 f = 0$)