MAT 361 Exam III **Name** _____

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate.
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space or the attached blank sheet. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use. Do not spend too much time on one problem. **Pace yourself.**

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	**17**	
2	**12**	
3	**9**	
4	**8**	
5	**9**	
Total	**55**	

Bonus: What is the amount of pure alcohol in each tank as $t \to \infty$?
[A little thought and few computations are needed.]

Bonus: Compute $e^{t\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}$.

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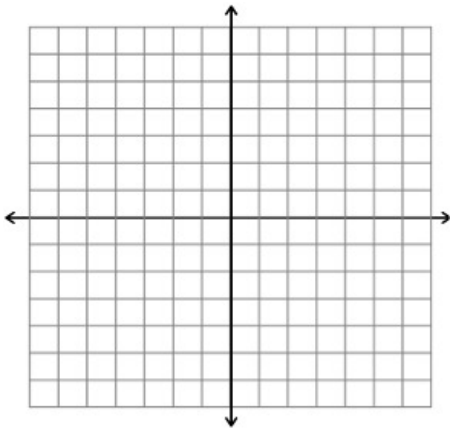
1. (12 pts) Find the following Laplace transforms. [No integration is necessary!]
   a. $L[te^{-2t}] =$

   b. $L[e^{-2t}\cos 3t] =$

   c. $L\left[\int_0^t (t-u)^2 \sinh 2u \, du\right] =$

   d. $f(t) = \begin{cases} \cos 2t, & 0 \le t \le \pi \\ 0, & \text{otherwise} \end{cases}$. $L[f(t)] =$

2. (5 pts) Consider the system $\begin{cases} x' = -2y \\ y' = 2x \end{cases}$. Use $\frac{dy}{dx}$ to find the curves $y = y(x)$.
   Draw at least three directed solution curves in the *xy* phase plane. What type of equilibrium point is the origin? _____



3. (9 pts) Find the following inverse Laplace transforms.

   a. $L^{-1}\left[\dfrac{s-1}{(s-3)(s+1)}\right]=$

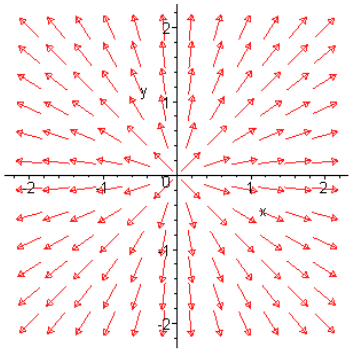
   b. $L^{-1}\left[\dfrac{e^{-s}}{s^2+9}\right]=$

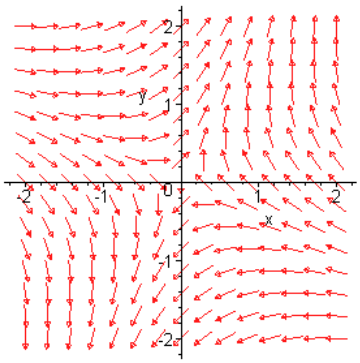
   c. $L^{-1}\left[\dfrac{s}{s^2-4s+9}\right]=$

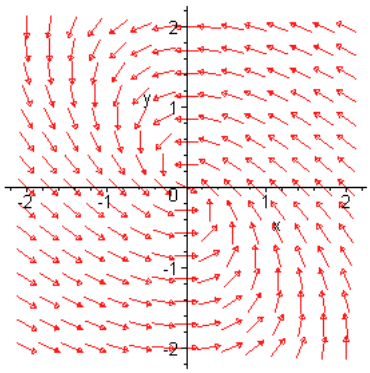
4. (3 pts) Use the Convolution Theorem to evaluate $L^{-1}\left[\dfrac{1}{s^2(s^2+1)}\right]$

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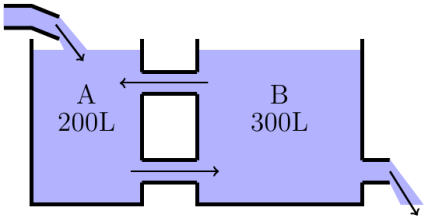
5. (9 pts) Describe the following phase plane plots by a) sketching solutions, b) discussing eigenvalues (positive/negative, real/complex), and c) classifying equilibrium points (saddle, stable node …).







6. (5 pts) Two tanks have a mixture of water and alcohol. Tank A has 200 L of pure water and tank B has 300 L of 90% concentration of alcohol. A 10% alcohol mixture enters tank A at a rate of 10 L/min and tank B drains at 10 L/min. The flows between the tanks include mixtures flowing from A to B at 15 L/min and from B to A at 5 L/min. Write down the differential equations governing the rates of change of the amount of pure alcohol in each tank and the initial conditions.



7. (3 pts) Solve using the Laplace transform method:

$$y''-4y'+4y=\delta(t-1),\ y(0)=0,\ y'(0)=1.$$

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8. (9 pts) Consider the system of differential equations: $\begin{cases} x' = x + 2y \\ y' = 2x + y \end{cases}$.

   a. Write the system in matrix form.

   b. Solve the eigenvalue problem.

   c. Give the Fundamental Solution Matrix and the general solution.

   d. What is the solution satisfying $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$?

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Extra Space