

Instructions:

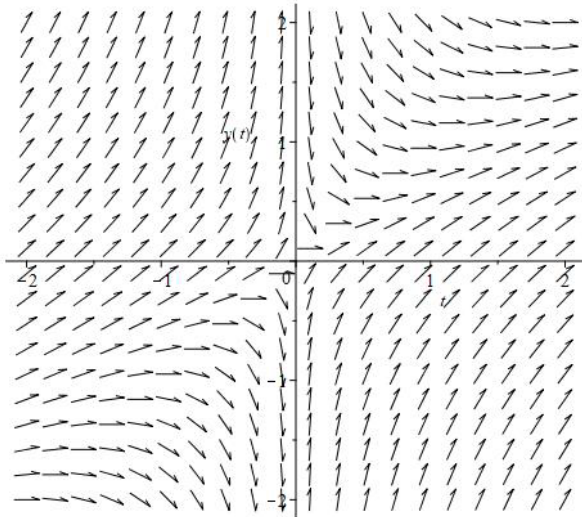
1. Do all of your work on this sheet.
2. **Show all of your steps** in problems for full credit.
3. **Be clear and neat** in your work. Any illegible work, or scribbling in the margins, will not be graded.
4. Place your **answers in a box**.
5. If you need more space, you may use the back page and write **On back** in the problem space.
6. All answers should be exact!

1. (13 pts) Answer the following:

- a. Identify all ordinary, regular singular, and irregular singular points for the equation:

$$x^2(1+x)^2 y'' - 3(1+x)y' + 5xy = 0.$$

- b. Sketch the solution curves passing through the points i. $y(0.5) = 2$ and ii. $y(-2) = -1$.



- c. Indicate the form you would guess for the particular solution:

i. $y'' - 4y = 3e^{-2x}$.

ii. $2y'' + 2y' + 1 = e^{-2x}$.

iii. $y'' + 4y = 3\cos x + 5x$.

iv. $x^2 y'' + 5xy' + 4y = x^3$.

Column**Score**

1	13 pts	
2	10 pts	
3	13 pts	
4	14 pts	
Total	50 pts	

2. (6 pts) Solve the differential equations for $x > 0$:

a. $x^2 y'' - 5xy' + 8y = 0$.

b. $x^2 y'' + 3xy' + 2 = x^2$.

3. (4 pts) Consider the problem $y' = xy$, $y(0) = 3$.
Use Euler's Method to approximate $y(1)$ using a step size of $h = 0.25$.

Bonus Find the truncation error in Problem 3.

4. (4 pts) Find the general solution of the equation $y'' - 3y' + 2y = 4x^2$ using the Method of Undetermined Coefficients

5. (4 pts) Use the Method of Variation of Parameters to find a particular solution of $y'' + y = \cot x$.

5. (5 pts) Consider the problem: $y' - xy = 0$, $y(0) = 3$. Use the Taylor Series Method to approximate the solution by a fourth degree polynomial in x . Use this polynomial to estimate $y(1)$.

6. (5 pts) Find the first four nonzero terms in a series solution of the Airy equation: $y'' - xy = 0$.

7. (4 pts) Solve $x'' + x = 2 \sin t$.

8. (5 pts) Consider the differential equation, $4xy'' + 2y' + y = 0$.

a. Find the indicial equation.

b. What are the roots of the indicial equation?

c. Explain how the Method of Frobenius uses this information.

Extra Space