MAT 361 Exam I **Name** _____

Instructions:
1. Do all of your work on this sheet.
2. **Show all of your steps** in problems for full credit.
3. **Be clear and neat** in your work. Any illegible work, or scribbling in the margins, will not be graded.
4. Place your **answers in a box**.
5. If you need more space, you may use the back of the page and write **On back** in the problem space.
6. All answers should be exact!

Column	Score
1 13 pts	
2 15 pts	
3 11 pts	
4 11 pts	
Total 50 pts	

1. (6 pts) Quickies – answer the following:
   a. Give two properties of a linear operator L:
      i.

      ii.

   b. What is orthogonal?

   c. Write $e^{(2-3i)x}$ in terms of trigonometric functions.

2. (7 pts) Find the general solution to the given differential equation. When an initial condition is given, find the particular solution.

   a. $y'+(\cos x)y^2=0,\ y(0)=2.$

   b. $\dfrac{dy}{dx}+\dfrac{1}{x}y=e^{-x}.$

3. (12 pts) Find the general solution to the given differential equations. When an initial condition is given, find the particular solution.
   a. $y''-2y'+5y=0$ for $y=y(x).$

   b. $x''+8x'+16x=0, x(0)=1, x'(0)=0,$ for $x=x(t).$

   c. $2y''+5y'+3y=0,\ y=y(x).$

4. (3 pts) Show that the functions $\cos 2x$ and $3\sin 2x-\cos 2x$ are linearly independent.

5. (5 pts) Determine if $(x - y\sin x)\,dx + \cos x\,dy = 0$ is exact, then find the solution.

6. (6 pts) Consider the differential equation
$$x^2 y'' + xy' - y = 0,\ \ x > 0\,.$$
a. Verify that $y(x) = x$ is a solution.

b. Find a second linearly independent solution using the method of reduction of order.

c. Based on your results, write the general solution.

7. (3 pts) The population of a town doubles after 30 years. How many years will it be before the population triples?

8. (5 pts) A thermometer at 18°F is brought into a 70°F room. One minute later the thermometer reads 31°F.
a. Find the temperature as a function of time.

b. What is the temperature reading 5 minutes after it was first brought into the room?

9. (3 pts) A 60.0 g mass is attached to a spring with spring constant 12.0 N/m. What damping constant is needed for critically damped motion?

Bonus: A ball falls according to $\dfrac{dv}{dt} = -9.8 - 0.49v$. What is the terminal velocity?