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Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate.
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space or the attached blank sheets. **No other scratch paper is allowed.**

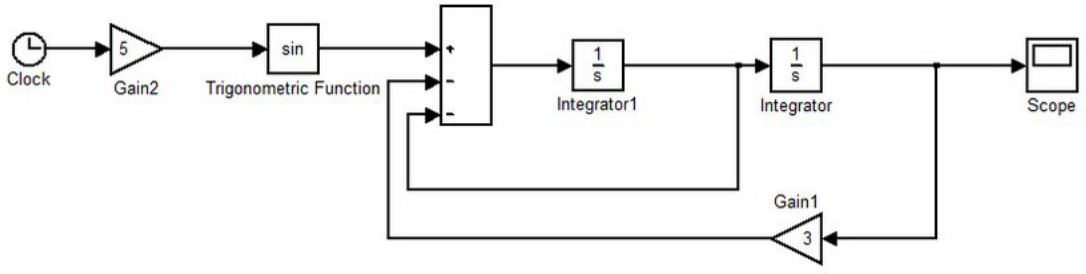
Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use. Do not spend too much time on one problem. **Pace yourself.**

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	18	
2	16	
3	16	
4	14	
5	11	
6	13	
7	12	
Total	100	

Bonus: What problem does this solve?



Have a good summer!

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1. (18 pts) Solve the following problems. If an initial condition is given, find the particular solution.

   a. $\frac{dy}{dx} = x + xy^2$

   b. $xy' - 3y = x^5 e^x,\ y(1) = 1.$

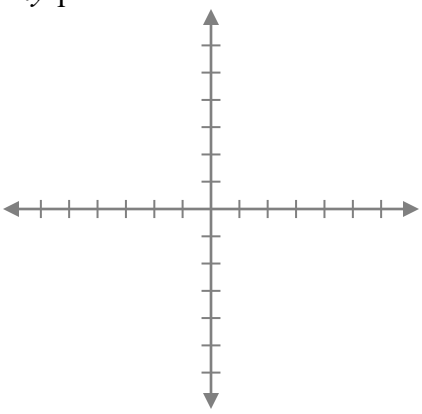
   c. $x'' + 3x' - 4x = 0.$

   d. $x^2 y'' + 5xy' + 16y = 0,\ y(1) = 2,\ y'(1) = 0.$

   e. $(y\cos x + 2xe^y)dx + (\sin x + x^2 e^y - 1)dy = 0.$

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2. (4 pts) Find and classify the equilibrium solutions (stable-unstable) of the equation $\frac{dy}{dt} = y^2 + y - 6$. Sketch these and other solutions throughout the *ty*-plane.



3. (8 pts) Solve the following nonhomogeneous problems:
   a. $y'' + 4y = 4\cos 2x$.

   b. $x^2 y'' - 4xy' + 4y = x^2$. Use the Method of Variation of Parameters.

4. (4 pts) A corpse is found at 3:00 P.M. in a 60°F room. The corpse was 79°F when discovered and was 68°F three hours later. Considering that normal body temperature is 98.6°F, determine the time of death.

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5. (4 pts) At UNCW there are 18,000 students. Five students decided to spread a rumor on campus. After one week, five hundred knew the rumor. Assume that the rate of spread of a rumor is proportional to the product of the number who know and the number who don't know the rumor, $y(N-y)$, where *y(t)* is the number who know the rumor at time *t*.
   a. Write down the differential equation for this model.

   b. How long would it take for half of the students to know the rumor?

6. (6 pts) Find the Laplace transform:
   a. $L\left[e^{-t}\sin 3t\right] =$

   b. $L\left[e^{-3t}+t^2e^{-t}\right] =$

7. (6 pts) Find the inverse Laplace transform:
   a. $L^{-1}\left[\dfrac{s}{(s-1)(s+3)}\right] =$

   b. $L^{-1}\left[\dfrac{1}{s^2+4s+6}\right] =$

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8. (10 pts) Consider the system of differential equations: $\begin{cases} x' = 2x + 3y \\ y' = 4x + 3y \end{cases}$.

   a. Convert the system to one second order differential equation.

   b. Determine the eigenvalues and eigenvectors of the coefficient matrix.

   c. Use your answer in part b to write the fundamental solution matrix.

   d. What is the particular solution satisfying $\mathbf{x}_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

9. (4 pts) Use the Method of Reduction of Order to find the second linearly independent solution of $4x^2 y'' + y = 0$, given $y_1(x) = \sqrt{x}$.

10. (7 pts) Consider the initial value problem $y' = y - x, \;\; y(0) = 2.$

    a. Use the Taylor series Method to find the first four terms in a series expansion of the form $y(x) = \sum_{n=0}^{\infty} c_n x^n$. Use this to get an approximation to $y(0.6)$.

    b. Use Euler's method to find an approximate value of $y(0.6)$, using $h = 0.2$ and $y(0) = 2$.

11. (4 pts) Find the first four terms of the power series solution, $y(x) = \sum_{n=0}^{\infty} c_n x^n,$ of $y'' - xy' + y = 0.$

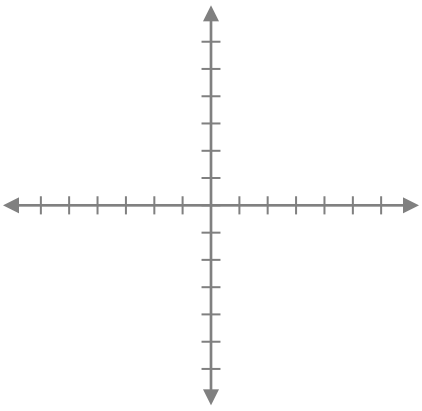
12. (5 pts) Solve $y''+2y'-3y=2\delta(t-1),\ y(0)=1,\ y'(0)=0,$ using the Laplace transform.

13. (3 pts) For the given points, determine if they are ordinary, regular singular or irregular singular points of the differential equation:
$x^2(x-1)y''-(x+2)y'+(x-1)y=0.$
    a. $x=0.$
    b. $x=1.$
    c. $x=2.$

14. (5 pts) Consider the system $\begin{aligned} x' &= -2x-xy+2x^2, \\ y' &= 5y+xy-2y^2. \end{aligned}$ Find and classify the equilibrium solutions.

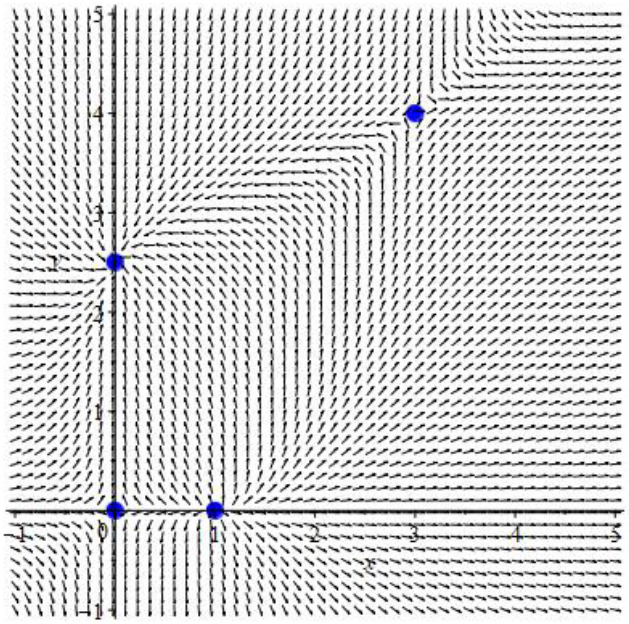
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15. (4 pts) Construct a bifurcation diagram for $y' = \mu y + y^2$.



16. (3 pts) Consider the convolution products $f(t) = t * \sin t$ and $g(t) = t^2 * \cos t$. Show that $G(s) = 2F(s)$ and, therefore, $g(t) = 2f(t)$.

17. (5 pts) Describe whatever you can about the following phase plane plot by sketching solutions, discussing eigenvalues and types of equilibrium points, noted in the figure.



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Extra Page