

**Instructions:**

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- All short answers and essays should be responded to with full sentences conveying thoughtful responses.
- If you need more space, you may use the **back of a page** and write *On back of page #* in the problem space or the **extra space** on the last page. **No other paper is allowed.**

**Try to answer as many problems as possible.** Provide as much information as possible. Show sufficient rationale for full credit when needed.

**Pay attention to the point distribution.** Not all problems have the same weight. Pace yourself!

| Page         | Pts        | Score |
|--------------|------------|-------|
| 1            | 34         |       |
| 2            | 17         |       |
| 3            | 13         |       |
| 4            | 20         |       |
| 5            | 16         |       |
| <b>Total</b> | <b>100</b> |       |

**Bonus:**

a. In Pascal's triangle, what is the 5<sup>th</sup> number in the 13<sup>th</sup> row?

b. Let  $x = \sqrt{16 + 6\sqrt{16 + 6\sqrt{16 + 6\sqrt{16 + 6\sqrt{\dots}}}}}$ . What is  $x$ ?

1. (8 pts) Match the Mathematician with the city.

Mersenne \_\_\_\_\_ Archimedes \_\_\_\_\_ Hypatia \_\_\_\_\_ Pythagoras \_\_\_\_\_  
 Fibonacci \_\_\_\_\_ da Vinci \_\_\_\_\_ Ferrari \_\_\_\_\_ Hippocrates \_\_\_\_\_

a. Samos b. Milan. c. Alexandria d. Chios e. Pisa. f. Florence g. Syracuse h. Paris

2. (12 pts) Select the approximate year for each mathematician/object.

Euclid \_\_\_\_\_ Archimedes \_\_\_\_\_ Cardano \_\_\_\_\_ Pythagoras \_\_\_\_\_  
 Aryabhata \_\_\_\_\_ Rhind Papyrus \_\_\_\_\_ Al-Khwarizmi \_\_\_\_\_ Lui Hui \_\_\_\_\_  
 Pacioli \_\_\_\_\_ Plimpton Tablet \_\_\_\_\_ Hypatia \_\_\_\_\_ Khayyam \_\_\_\_\_

a. 1900-1800 BCE b. 1700-1600 BCE c. 550-450 BCE d. 350-250 BCE e. 300-200 BCE  
 f. 200-300 g. 300-450 h. 500-600 i. 800-900 j. 1000-1100 k. 1100-1300. l. 1400-1550.

3. (10 pts) Can you name that mathematician? [Hint: The names have been used above.]



A B C D E F G H I J

A. \_\_\_\_\_ E. \_\_\_\_\_ I. \_\_\_\_\_

B. \_\_\_\_\_ F. \_\_\_\_\_ J. \_\_\_\_\_

C. \_\_\_\_\_ G. \_\_\_\_\_

D. \_\_\_\_\_ H. \_\_\_\_\_

4. (4 pts) Consider the Babylonian number below (spaces are not zeros).



a. Write this as a sexagesimal number using slash (/) notation. \_\_\_\_\_

b. Write it in the base 10 system. \_\_\_\_\_

5. (5 pts) Answer the following questions by filling in the blank.

- a. Who is said to be the founder of algebra? \_\_\_\_\_
- b. What do we call the numbers we use today? \_\_\_\_\_
- c. What is the golden ratio? \_\_\_\_\_
- d. What's another name for Euclid's 5<sup>th</sup> postulate? \_\_\_\_\_
- e. What are the numbers 1,3,6,10, ... called? \_\_\_\_\_

6. (4 pts) Write the following as sexagesimal numbers.

- a. 1003

b.  $\frac{2}{7} = 0.\overline{285714}.$

7. (6 pts) Consider the numbers 225 and 81.

- a. Use the Euclidean algorithm to find the greatest common divisor.

- b. Solve  $225m + 81n = 9$  for  $m$  and  $n$  integers.

8. (2 pts) Can one construct a heptagon (regular 17-gon)? \_\_\_\_\_

Who was the first to answer this question? \_\_\_\_\_

9. (3 pts) Consider the prime number  $2^{13} - 1$ .

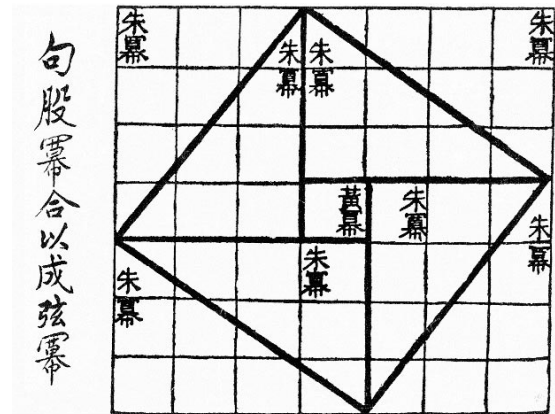
- What is this prime called?
- Use this number to create a perfect number.

10. (5 pts) Zhao Shuang added a commentary to the text, *Zhoubi Suanjing*, and added the below diagram which demonstrates the Guogu Theorem, or Rule.

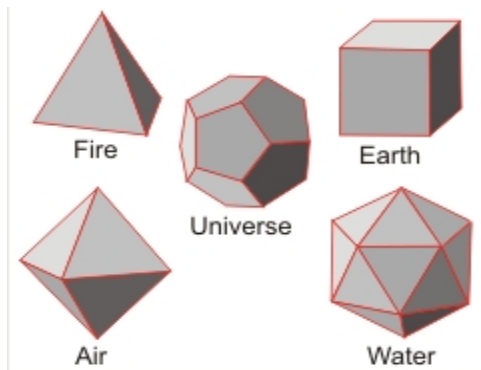
- Specifically, what are these terms called?

Gou \_\_\_\_\_ Gu \_\_\_\_\_ Xian \_\_\_\_\_

- Explain this diagram.



11. (5 pts) Plato identified the solids below with the properties shown. What are their names?



Fire \_\_\_\_\_

Earth \_\_\_\_\_

Air \_\_\_\_\_

Universe \_\_\_\_\_

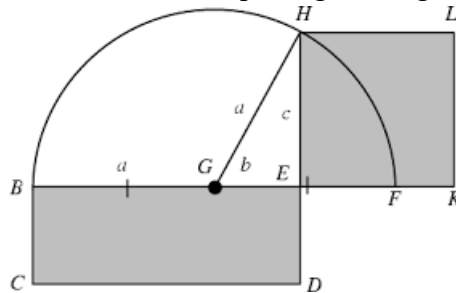
Water \_\_\_\_\_

12. (6 pts) Answer the following about the *Zhoubi Suanjing* and *Jiuzhang Suanshu*.
- When were they believed to be written? \_\_\_\_\_
  - Which one is The Nine Chapters of the Mathematical Art?  
\_\_\_\_\_
  - What three major pieces of mathematics are found in the Nine Chapters that predated European mathematics? \_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_.
  - The other work is translated as “Arithmetical Classic of the Gnomon and the Circular Paths of Heaven.” What is a gnomon? \_\_\_\_\_.
13. (4 pts) Find two pairs of integer solutions of  $x^2 - 8y^2 = 1$ . [Hint: Start with  $y = 1$ .]
14. (10 pts) Name the most important contribution to mathematics by these people in a couple of words:
- Apollonius of Perga \_\_\_\_\_
  - Brahmagupta \_\_\_\_\_
  - del Ferro \_\_\_\_\_
  - Eratosthenes \_\_\_\_\_
  - Viète \_\_\_\_\_
  - Liu Hui \_\_\_\_\_
  - Pacioli \_\_\_\_\_
  - Leonardo of Pisa \_\_\_\_\_
  - Madhava \_\_\_\_\_
  - Bombelli \_\_\_\_\_

15. (7 pts) What do you know about solving the cubic?

- What Persian mathematician first classified cubic equations? \_\_\_\_\_
- Give an example of a “depressed cubic” equation: \_\_\_\_\_
- Name in order the first three mathematicians who knew how to solve this cubic.  
\_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_
- Who used the solution of the cubic to solve the quartic? \_\_\_\_\_
- How is doubling the cube related to solving the cubic?

16. (9 pts) The figure below is the construction for squaring rectangle BCDE.



- Why is this called a construction?
- What does it mean to square a rectangle?
- Show why the construction is the quadrature of the rectangle ABCD using the appropriate line segments and lengths  $a = \overline{BG} = \overline{GH}$ ,  $b = \overline{GE}$ , and  $c = \overline{HI}$ .

**Extra Space**