

Introduction to the History of Mathematics

Fall 2020 - R. L. Herman



Table of Contents

Early Civilizations - Babylonia, Egypt, China, India, Islamic

Beyond Numerals - Decimals, Logarithms, Symbolic Algebra

Italian Mathematics - 16th Century

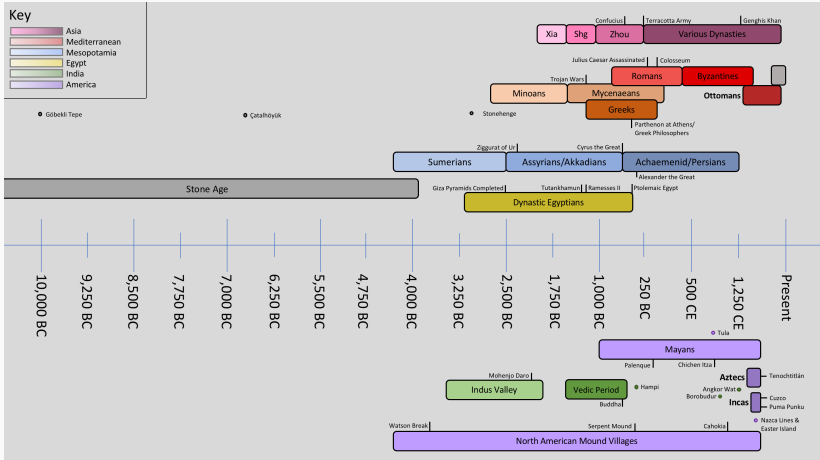
The Rise of Calculus - 17th Century

Exploiting Calculus - 18th Century

The Birth of Rigor - 19th Century

The Modern Era - 20th Century

A Civilization Timeline



Early Civilizations

- Egypt - 3100 BCE
- Mesopotamia (Babylonia, Sumer) - 2100 BCE
- China 1600 BCE
- India 1200 BCE

Arithmetic, Geometry,

No proofs

Problems were practical or recreational



$$(60)^3 + 11(60)^2 + (50 - 3)(60) + 40 - 2 =$$
$$(60)^3 + 11(60)^2 + 47(60) + 38 = 258,458$$

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Figure 1: Babylonian tablet - Base 60

Greek Civilization

- Deductive Reasoning
 - Definitions, Axioms
 - Propositions via logic
- Geometry, Trigonometry, Astronomy, Numbers, Conics
- Thales 624-546 BCE
- Pythagoras 6th Century BCE
- Euclid 4th Century BCE
 - Elements*
- Archimedes 3rd Century BCE
- Appolonius 2nd Century BCE
- Heron 10-70 CE
- Diophantus 200-284 CE,
- Hypatia 400 CE



Figure 2: Euclid

Chinese and Hindu - 700-1200 CE

- Chinese Mathematics 1300 BCE - 1800 CE
 - Pythagorean Thm
 - π estimates
 - Volumes, Applications
 - Pascal's Triangle
 - Chinese Remainder Thm
- Indian Mathematics 1200 BCE, mostly 500-1200 CE
 - Geometry
 - Trigonometry
 - Power series
 - Astronomy
 - π estimate
 - Number system, 0
 - Pell's Equation

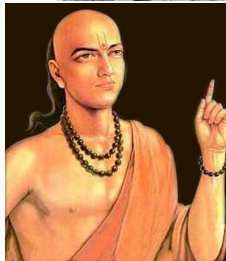


Figure 3: Liu-Hong and Aryabhata

Middle Eastern Mathematics - 700-1200 CE

In Europe - Dark Ages - 400-1200 CE

- Middle East - Islam 7th Century
- Translations of Greek Mathematics into Arabic
- Arabic Numerals by 1000 CE
- Algebra 825 CE - al-Khwarizmi - called al-Jabr
- Omar Khayyam (1048-1131) - geometric solution of cubic

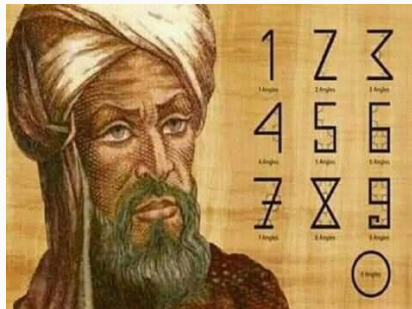


Figure 4: al-Khwarizmi

Around 10th Century - Middle Eastern Mathematics brought to Spain.

It takes 300 years to accept Arabic numerals. - Fibonacci - 1202

- Fibonacci (Leonardo of Pisa)
(1170-1250) *Liber Abaci*
- Equation Solving contests
- Solutions of cubic and quartic
- Depressed cubic
del Ferro (1465-1526)
- Cubic and quartic equations
Tartaglia (1500-1557)
Cardano (1501-1576)
Ars Magna
Ferrari (1522-1565)
- Bombelli (1526-1572)
- Viète (1540-1603)
Adriaan van Roomen Problem



Figure 6: Cardano and Tartaglia

Unification of Geometry and Algebra

- Symbolic Algebra
 - Rhetorical until 15th century
 - Syncopated/abbrev. - 1500
 - Symbolic algebra developed 16-17th century
- Unification
 - Oresme (1320-1382) -
Velocity-time graphs, $\sum \frac{1}{n}$
 - Descartes (1596-1650)
 - Rep. curves by equations
 - Coordinate systems -
published *The Method*
 - Made use of variables which
can vary continuously - lines.
 - Fermat (1607-1665)
 - Rep. equations by curves



Figure 7: Fermat and Descartes

The Rise of Calculus

- Archimedes - 3rd century BCE
- Kepler (1571-1630)
- Cavalieri (1598-1647)
- Fermat (1607-1665)
- Wallis (1616-1673)
- Pascal (1623-1662)
- Barrow (1630-1677)
- Wren (1632-1723)
- Gregory (1638-1675)
- Newton (1642-1726)
 - *Principia* 1687
- Leibniz (1646-1716)
 - Notation $\frac{d}{dx}$, \int

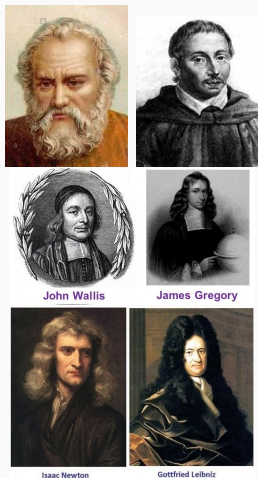


Figure 8: Archimedes, Cavalieri, Wallis, Gregory, Newton, and Leibniz

The Infinitesimal

- Hippasus 500 BCE
 - Pythagorean
 - $\sqrt{2}$ irrational
- Introduction of Infinitesimals
 - Cavilieri and Torricelli
 - Stevin, Wallis, Harriot
- Critics
 - Jesuits in Italy
 - George Berkeley (1685-1753)
The Analyst, - A Discourse Addressed to an Infidel Mathematician, 1734
 - infinitesimals undermine mathematics and rationality
- Augustin-Louis Cauchy - 1821

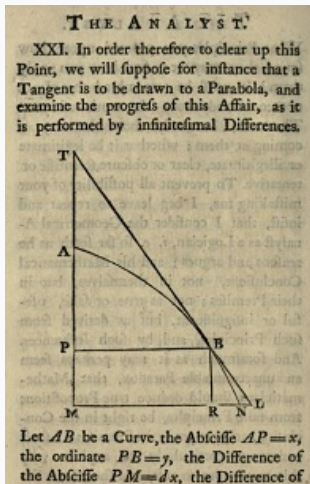
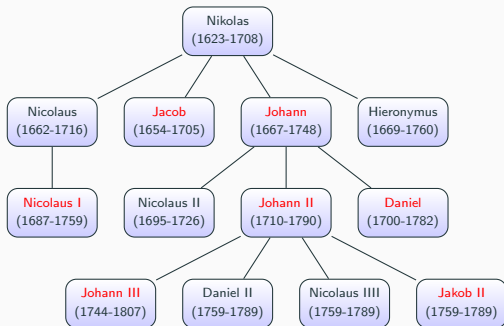
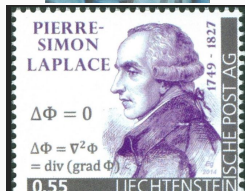


Figure 9: Berkeley's *The Analyst*

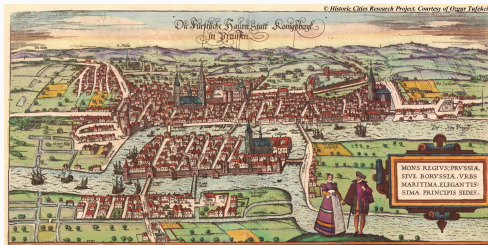
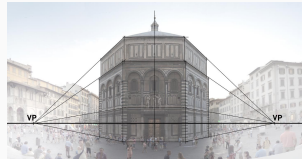
Exploiting Calculus

- Bernoulli Family
- Euler (1707-1783)
- Laplace (1749-1827)
- Neptune discovered using math
- 1846



From Geometry to Topology

- Bruneschelli and Perspective
- Projective Geometry
- Euler's geometry without distance
- Birth of Topology
- How smoke rings led to Knot Theory.



The Birth of Rigor - 19th Century

Non-Euclidean Geometry

- Parallel Postulate
- Hyperbolic Geometry
 - Nikolai Lobachevsky (1792-1856)
 - Johann Bolyai (1802-1860)
 - Johann Carl Friedrich Gauss (1777-1855)
- Elliptic Geometry
 - Georg Friedrich Bernhard Riemann (1826-1866)
 - Prince of Mathematicians
- By 1870's doubted Euclid



Figure 11: Gauss, Lobachevsky, Bolyai

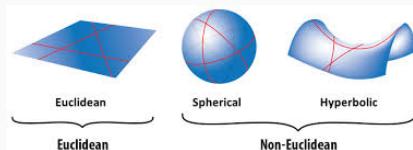


Figure 12: Different Geometries

19th Century Group Theory

- Joseph-Louis Lagrange (1736-1813)
- Johann Carl Friedrich Gauss (1777-1855)
- Augustin Cauchy (1789-1857)
- Niels Henrik Abel (1802-1829)
- Évariste Galois (1811-1832)
- Arthur Cayley (1821-1895)
- Camille Jordan (1838-1922)



Figure 13: Abel and Galois

19th Century Analysis and Set Theory

- Jean-Baptiste Joseph Fourier (1768-1830)
- Johann Carl Friedrich Gauss (1777-1855)
- Augustin Cauchy (1789-1857)
- Karl Weierstrass (1815-1897)
- George Boole (1815-1864)
- Georg Friedrich Bernhard Riemann (1826-1866)
- Richard Dedekind (1831-1916)
- Georg Ferdinand Ludwig Philipp Cantor (1845-1918)
 - Founder of set theory
 - Defined infinite sets



Figure 14: Gauss and Riemann

19th Century Number Theory

- Marie-Sophie Germain (1776-1831)
- Johann Carl Friedrich Gauss (1777-1855)
Disquisitiones Arithmeticae - 1801
- Adrien-Marie Legendre (1752-1833) and Peter Gustav Lejeune Dirichlet (1805-1859) prove Fermat's Last Theorem for $n = 5$ in 1825
 - Dirichlet, $n = 14$ in 1832.
- Riemann hypothesis, distribution of primes - 1832.
- Charles Jean de la Vallée-Poussin and Jacques Hadamard - Prime Number Theorem. 1896
- Hermann Minkowski: Geometry of Numbers, 1896.



Figure 15: Sophie Germain, Adrien-Marie Legendre

The Modern Era



Figure 16: Hilbert, Gödel, Uhlenbeck, Ramanujan, Wiles, Mirzakhani, Shannon, Russell, Noether

- Chronology of 20th Century Mathematicians
- Greatest Mathematicians born between 1860 and 1975
- Pictures of Famous 20th Century Mathematicians
- The Story of Math Website