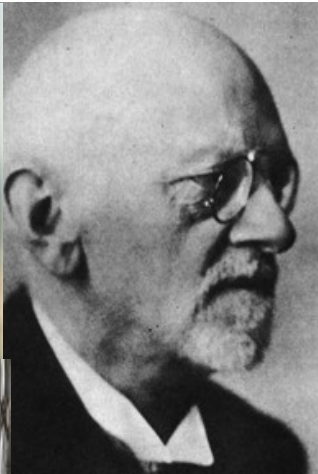
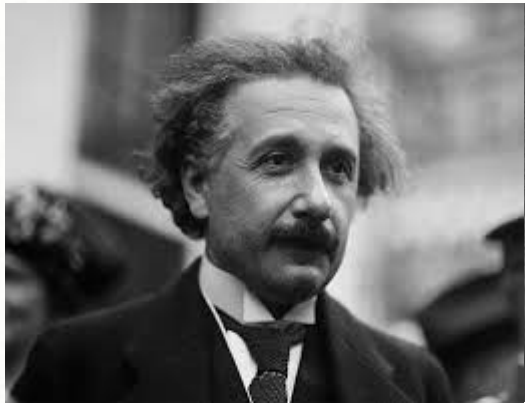


# *History of Mathematics - 1900s*

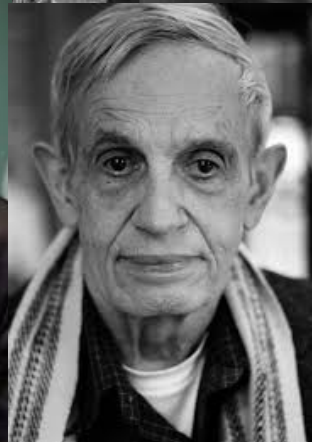
*Dr. R. L. Herman, UNCW*





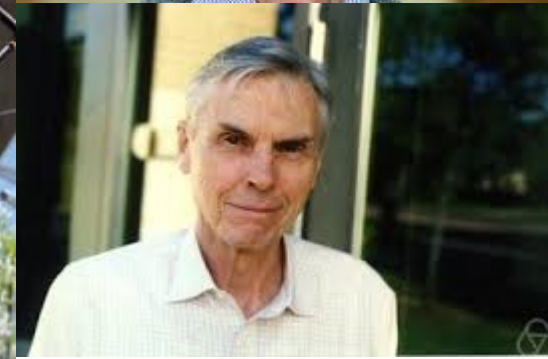
## *Outline*

- Evolution of Mathematics
- Set Theory
- Physics Revolutions
- Mathematics Prizes



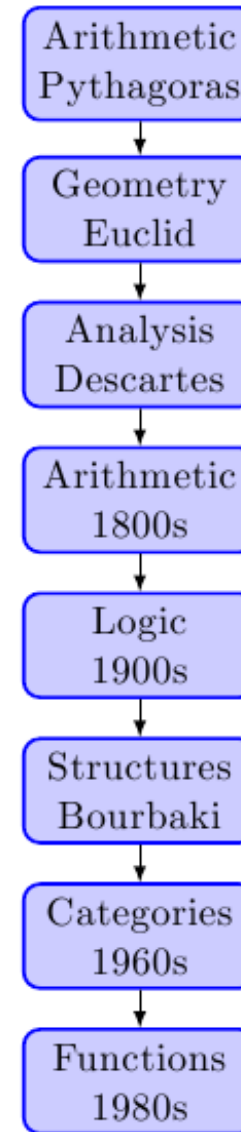
# *Mathematics Prizes*

- Fields Medal
- Abel Prize
- Wolf Prize
- Millenium Prize



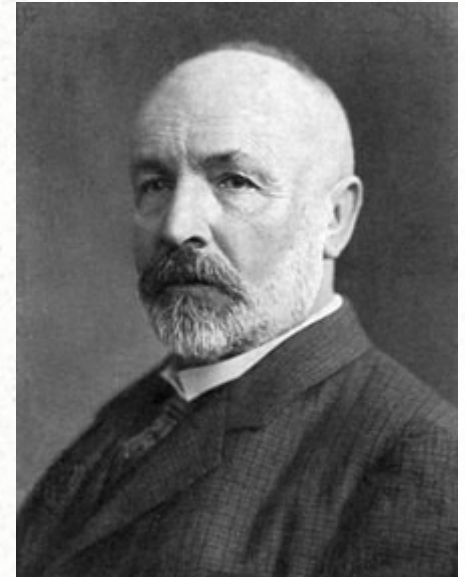
# *Evolution of Mathematics*

- Pythagoreans - Arithmetic
- Euclid - Geometry
- Descartes – Analytic Geometry
- Newton – Calculus, Mechanics
- Euler – Numbers, Applications
- Gauss – Noneuclidean geometry
- Cantor – Set Theory, Infinity
- Frege – Predicate Logic
- Russell – *Principia Mathematica*
- Gödel – Incompleteness Theorems



# *Georg Cantor (1845-1918)*

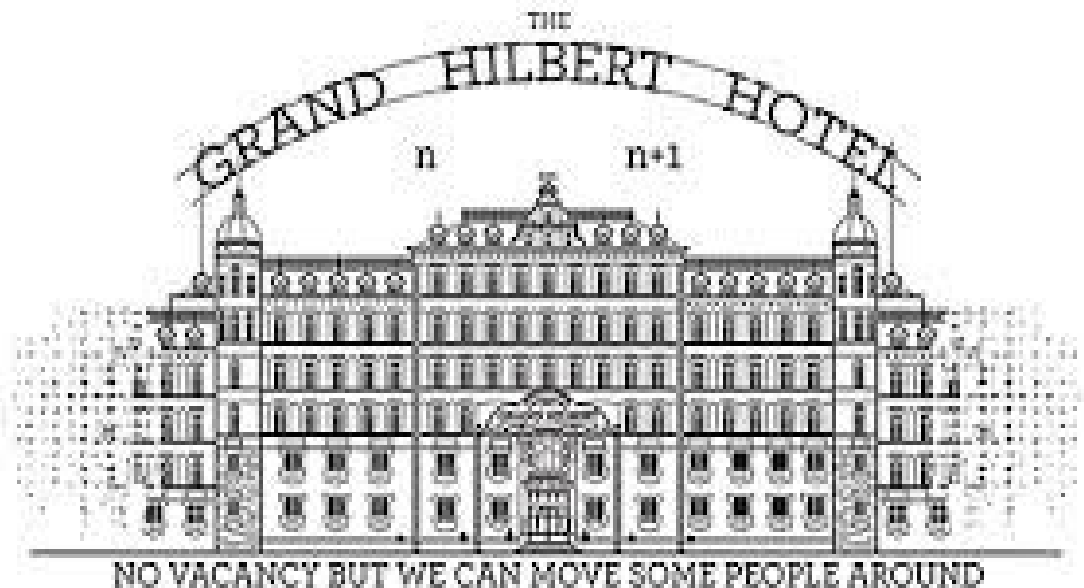
- Created set theory.
- Cardinality of sets.
- Not all infinities have same cardinality.
  - Natural Numbers, Rationals, Reals
- Transfinite numbers and the Continuum hypothesis



There are no intermediate cardinal numbers between  $\aleph_0$  (aleph-null) and the cardinality  $\aleph_1$  of the continuum (set of real numbers).

# *Hilbert's Grand Hotel - 1924*

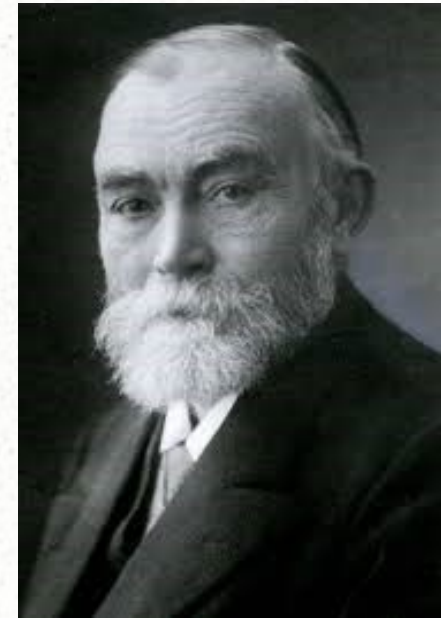
- David Hilbert (1862-1923)
- Infinite # rooms fully occupied with infinite # people.
- Add a person.



# *Gottlob Frege (1848-1925)*

Invented axiomatic predicate logic,  
Essential to

- Principia Mathematica (1910–13)  
Bertrand Russell, (1872–1970), and  
Alfred North Whitehead, (1861–1947).
- Kurt Gödel's (1906–78) incompleteness  
theorems.
- Alfred Tarski's (1901–83) theory of  
truth.
- Development of set theory.



# *Bertrand Russell, (1872–1970)*

- Philosopher, logician, mathematician, historian, writer, essayist, social critic, political activist, and Nobel laureate
- Liar's Paradox

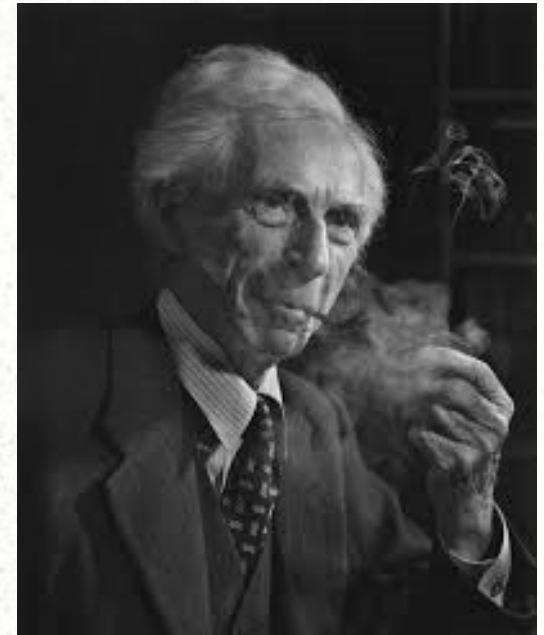
“This sentence is a lie”

- Russell's Paradox

Consider the set of all sets (power set) that are not members of themselves.

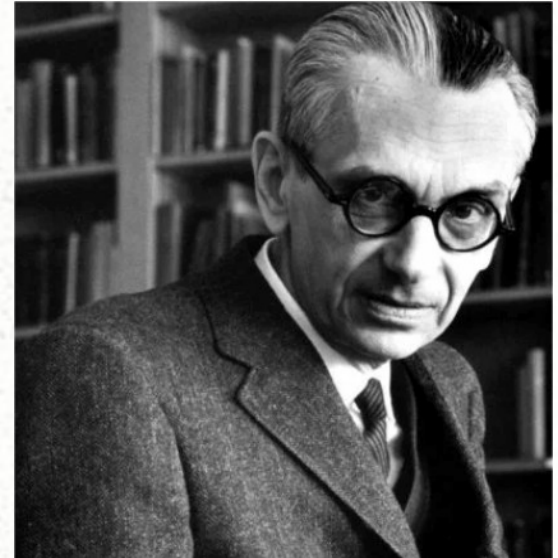
$$\text{Let } R = \{x \mid x \notin x\}, \text{ then } R \in R \iff R \notin R$$

- Avoid paradox - Zermelo–Fraenkel set theory



# *Kurt Gödel (1906–78)*

- 1930 Incompleteness Theorems
  1. If a (logical or axiomatic formal) system is consistent, it cannot be complete.
  2. The consistency of axioms cannot be proved within their own system.
- Met Einstein 1933.
- Moved to Princeton 1940.
- 1949 Rotating universes and time travel.





Henri Cartan



André Weil



Jean Dieudonné



Claude Chevalley



René de Possel



Charles Ehresmann



Pierre Samuel



Jean-Pierre Serre



Laurent Schwartz



Adrien Douady

# *Nicolas Bourbaki (1935 - )*

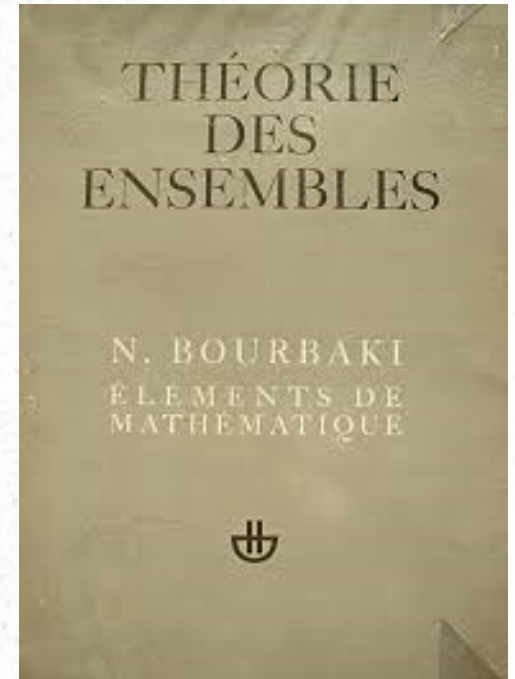
*École Normale Supérieure, Paris*

- Founders

- Henri Cartan,
- Claude Chevalley,
- Jean Coulomb,
- Jean Delsarte,
- Jean Dieudonné,
- Charles Ehresmann,
- René de Possel, Szolem Mandelbrojt,
- André Weil.

- Notable participants in later days:

- Schwartz, Serre, Grothendieck, Eilenberg, and Serge Lang.



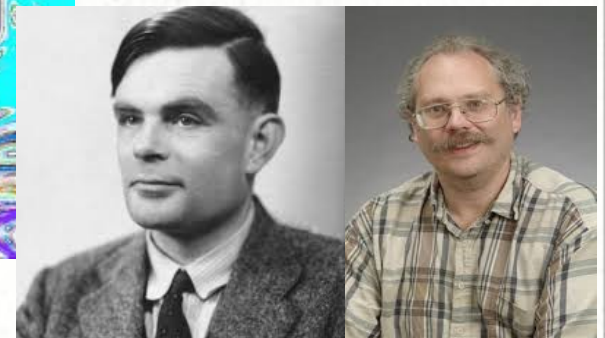
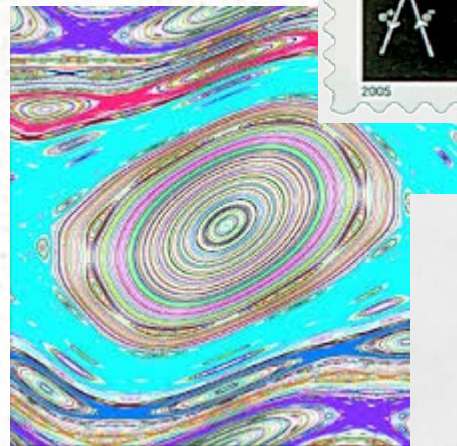
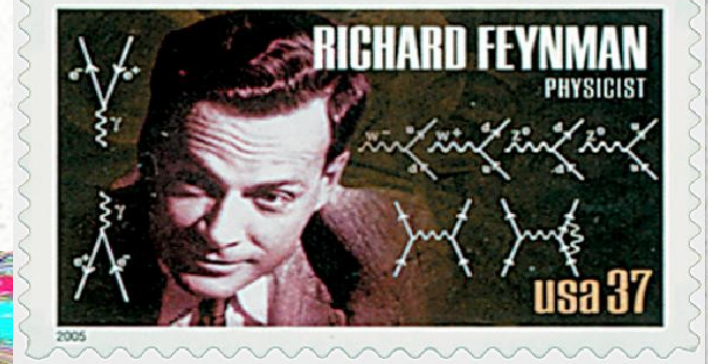
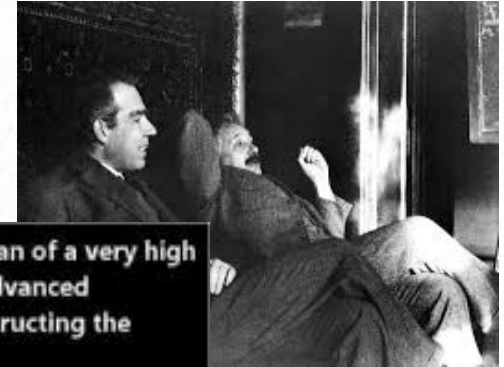
# *Revolutions*

- Quantum Theory
- Special Relativity
- General Relativity
- String Theory Revolution
- Nonlinear Dynamics and Chaos
- Information Age



"God is a mathematician of a very high order and He used advanced mathematics in constructing the universe."

-Nobel Prize winning physicist Paul A. M. Dirac, who made crucial early contributions to both quantum mechanics and quantum electrodynamics.

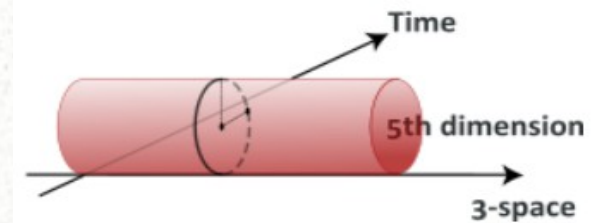


*Focus on higher dimensions ...*

# *Kaluza-Klein to Calabi-Yau*

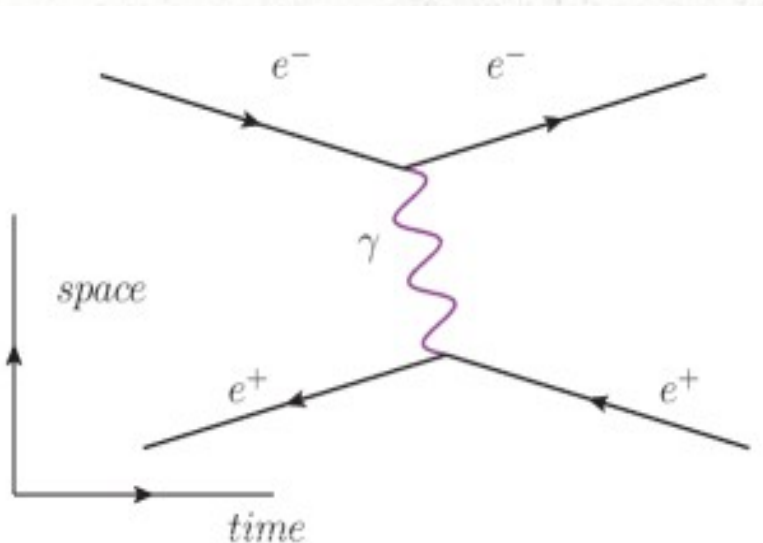
## *The Search for Higher Dimensions*

- The 4<sup>th</sup> Dimension
  - Special Relativity 1905;
  - Minkowski – 4D spacetime.
  - General Relativity 1915.
- Theodor Kaluza (1885-1954) In 1921
  - Solved Einstein Equations in 5D.
  - Unified General Relativity with E&M.
- Oskar Klein (1894-1977) – QM Interpretation
  - fifth dimension - curled up, microscopic
- Led to gauge theories on fiber bundles.



# *Feynman Diagrams and QED*

- Richard Feynman (1918-1988)
- Quantum Electrodynamics (QED) – 1948-9
  - Interaction of light and matter
  - Nobel Prize with Schwinger, Tomonaga 1965



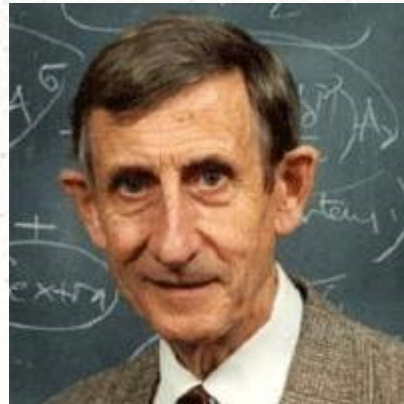
Not to F. Dyson



# *Freeman Dyson*

## *1972 Gibbs Lecture*

I am acutely aware of the fact that the marriage between mathematics and physics, which was so enormously fruitful in past centuries, has ended in divorce.

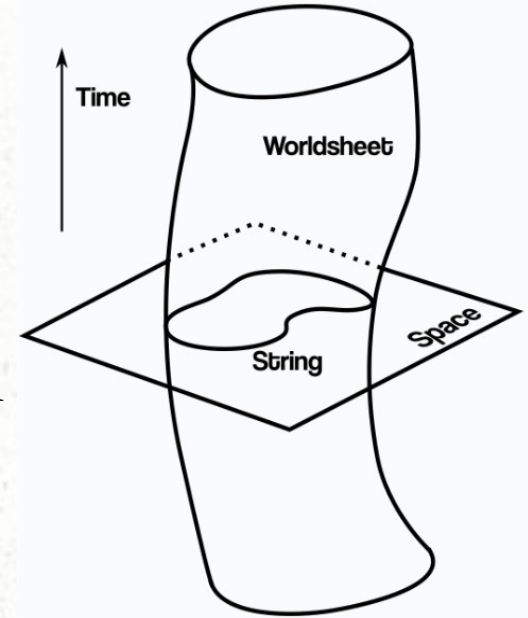


# String Theory

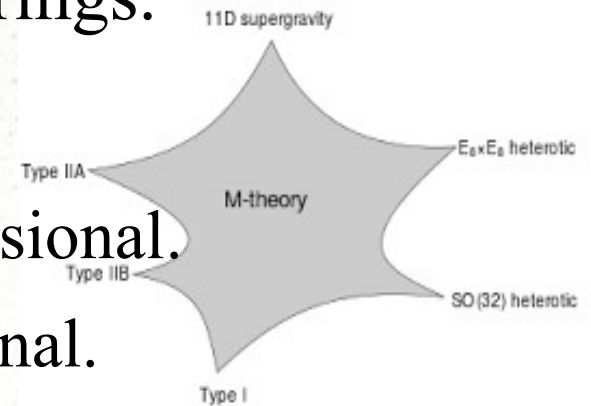
- Gabriele Veneziano (1942-)

In 1968 “Thumbing through old math books, they [Mahiko Suzuki] stumbled by chance on the [Euler] Beta function ...,” Michio Kaku

$$B(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} dt$$

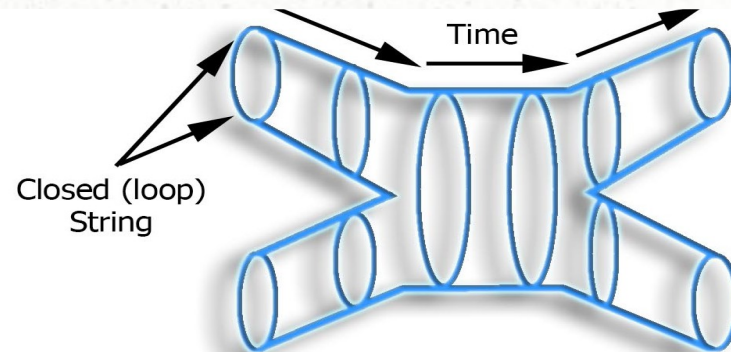
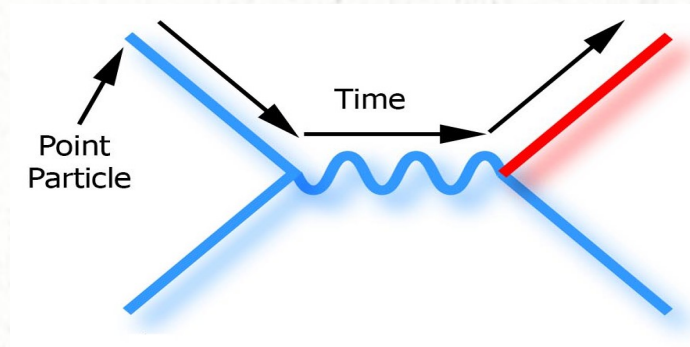


- Point-like particles modeled as 1D strings.
- Extra Dimensions 70s-90s
  - Bosonic string theory, 26-dimensional.
  - Superstring theory, 10-dimensional.
  - M-theory, 11-dimensional.

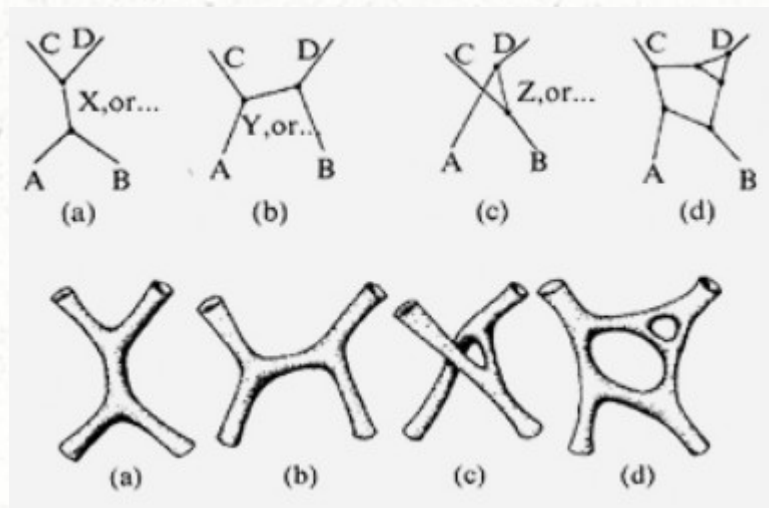


# *Feynman Diagrams to Strings*

- Extend Feynman diagrams



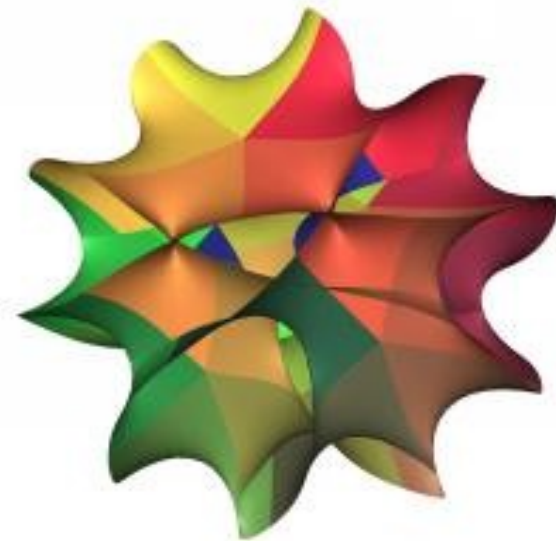
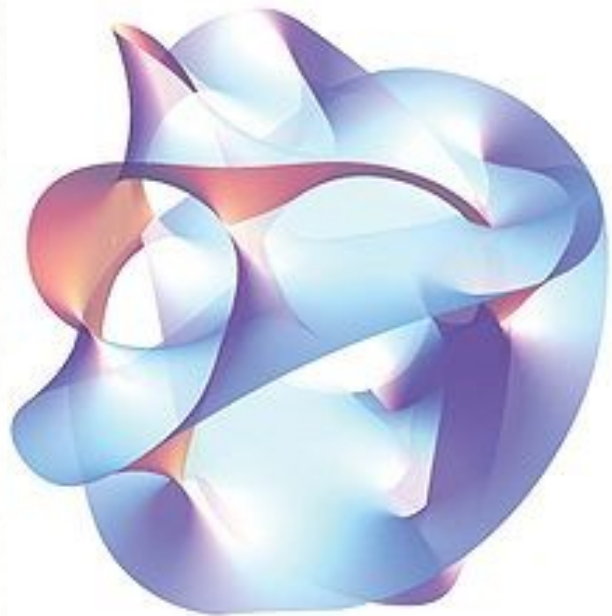
- Differential Geometry
- Topology
- Knot Theory



# *Calabi-Yau Manifolds*

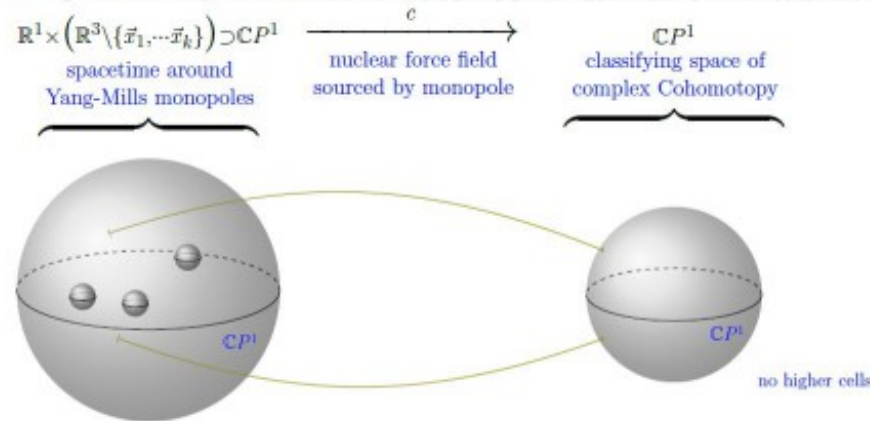
- Compactification (curl up extra 6 dimensions).
- Eugenio Calabi and Shing-Tung Yau  
Mathematicians

$$x_1^5 + x_2^5 + x_3^5 + x_4^5 + \psi x_1 x_2 x_3 x_4 = 1$$



# Mathematics and Physics

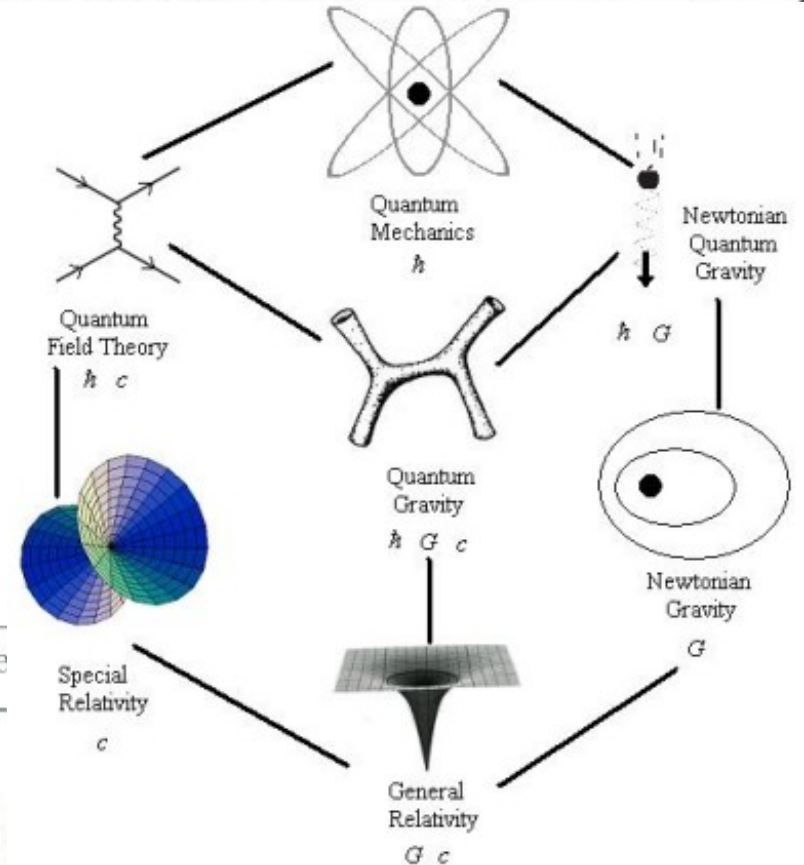
## - Not Divorced



$$[c] \in \{ \mathbb{C}P^1 \rightarrow \mathbb{C}P^1 \} / \sim_{\text{homotopy}} \simeq \mathbb{Z}$$

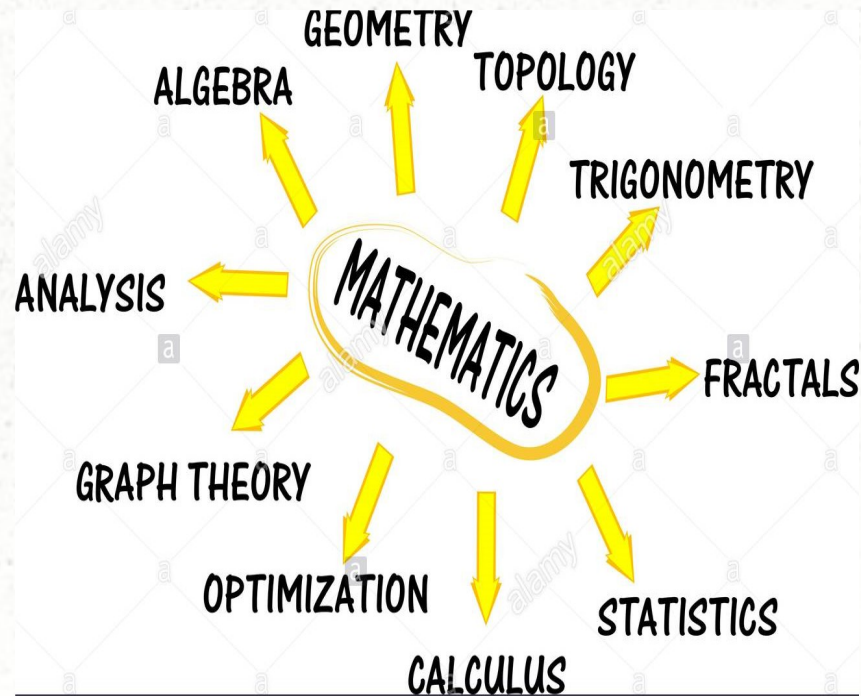
charge = homotopy class charge lattice

**Atiyah-Hitchin charge quantization** – The moduli space of SU(2) Yang-Mills monopoles is the cocycle space of complex-rational Cohomotopy of any sphere enclosing them.



Michael Atiyah (1929-2019)

# The Greatest Problems and Prizes



## *David Hilbert (1862-1943)*

- Opened the International Congress of Mathematicians in Paris in the year 1900.
- Outlined 23 major mathematical problems to provide solutions for in the coming new century.



# *Hilbert's Problems 1-6*

1. Cantor's problem of the cardinal number of the continuum. (partially resolved)
2. The compatibility of the arithmetic axioms.
3. The equality of two volumes of two tetrahedra of equal bases and equal altitudes. - (resolved)
4. Problem of the straight line as the shortest distance between two points. (vague)
5. Lie's concept of a continuous group of transformations without the assumption of the differentiability of the functions defining the group. (i.e., are continuous groups automatically differential groups?)
6. Mathematical treatment of the axioms of physics.

## *Hilbert's Problems 7-14*

7. Irrationality and transcendence of certain numbers.
8. Problems (with the distribution) of prime numbers.
9. Proof of the most general law of reciprocity in any number field.
10. Determination of the solvability of a diophantine equation.
11. Quadratic forms with any algebraic numerical coefficients.
12. Extension of Kronecker's theorem on abelian fields.
13. Impossibility of the solution of the general equation of the 7th degree.
14. Proof of the finiteness of certain complete systems of functions.

## *Hilbert's Problems 15-23*

15. Rigorous foundation of Schubert's calculus.
16. Problem of the topology of algebraic curves and surfaces.
17. Expression of definite forms by squares.
18. Building space from congruent polyhedra.
19. Are the solutions of regular problems in the calculus of variations always necessarily analytic?
20. The general problem of boundary curves.
21. Proof of the existence of linear differential equations having a prescribed monodromic group.
22. Uniformization of analytic relations by means of automorphic functions.
23. Further development of the methods of the calculus of variations.

# *Fields Medal*

Prize (John Charles Fields)

- 2-4 mathematicians under 40 yrs.
- The International Congress of the International Mathematical Union
- Every four years.



*Transire suum pectus mundoque potiri.*  
Rise above oneself and grasp the world.

See [Fields Medalists](#)



## *Abel Prize*



- 1899 Proposed by the Norwegian mathematician Sophus Lie (1842-1899).
- He learned of Alfred Nobel's plans.
- First Awarded 2003

See [Laureates](#)



Niels Henrik Abel (1802-1829)

# *Wolf Prize in Mathematics*

- Awarded almost annually by the Wolf Foundation, Israel.
- One of the 6 Wolf Prizes since 1978;
  - Agriculture, Chemistry, Medicine, Physics and Arts.
- See [Winners](#)

Greg Lawler  
2019



Charles Fefferman  
2017



George Mostow  
2013



Jean-François Le Gall  
2019



Richard Schoen  
2017



Michael Artin  
2013



Vladimir Drinfeld  
2018



James Arthur  
2015



Luis Caffarelli  
2012



Alexander Beilinson  
2018



Peter Sarnak  
2014



Michael Aschbacher  
2012



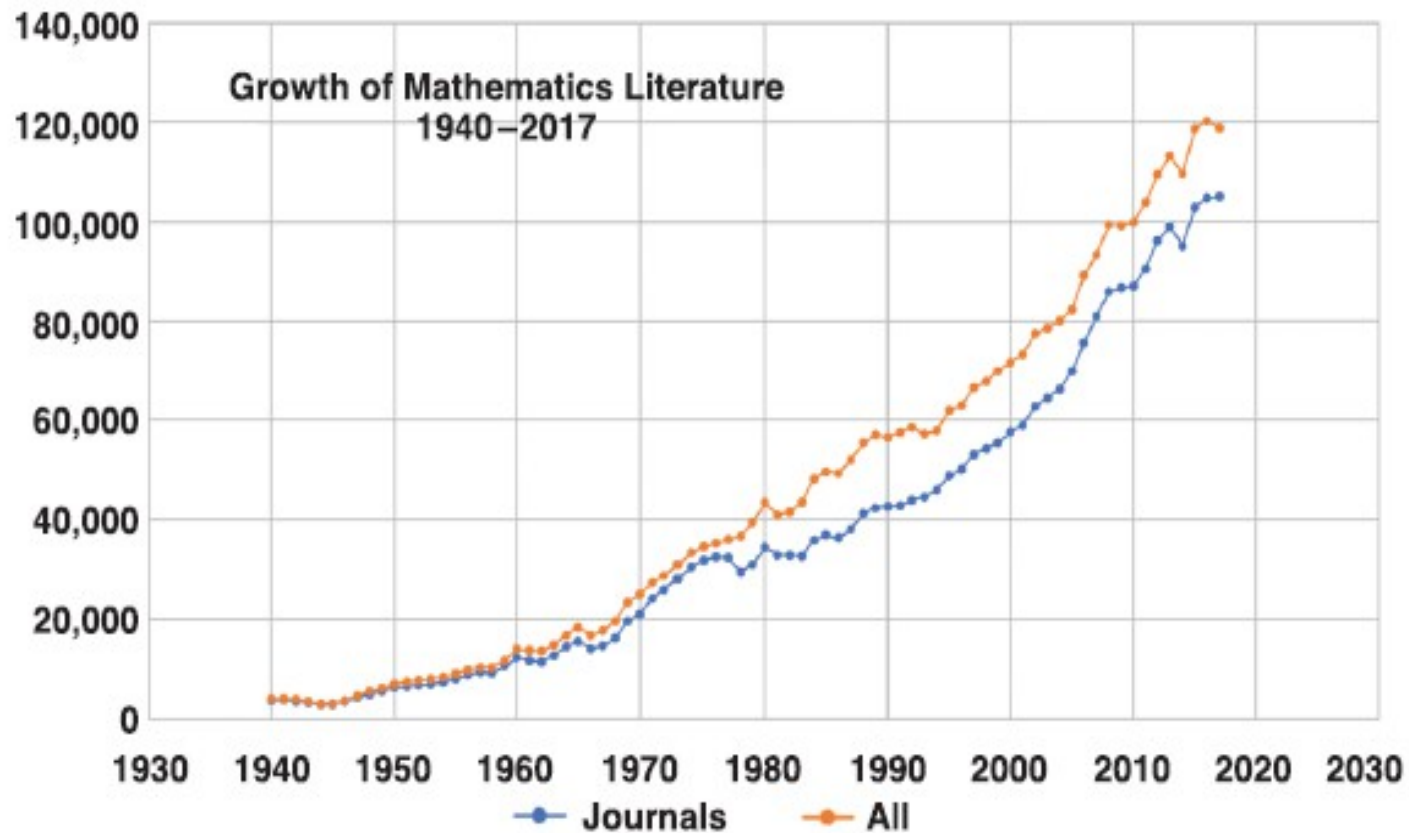
# *Millennium Prize Problems*

Stated by the Clay Mathematics Institute on May 24, 2000 – for One Million Dollars

- Birch and Swinnerton-Dyer conjecture,
- Hodge conjecture,
- Navier–Stokes existence and smoothness,
- P versus NP problem,
- Poincaré conjecture (**Solved! - Perelman**),
- Riemann hypothesis, and
- Yang–Mills existence and mass gap

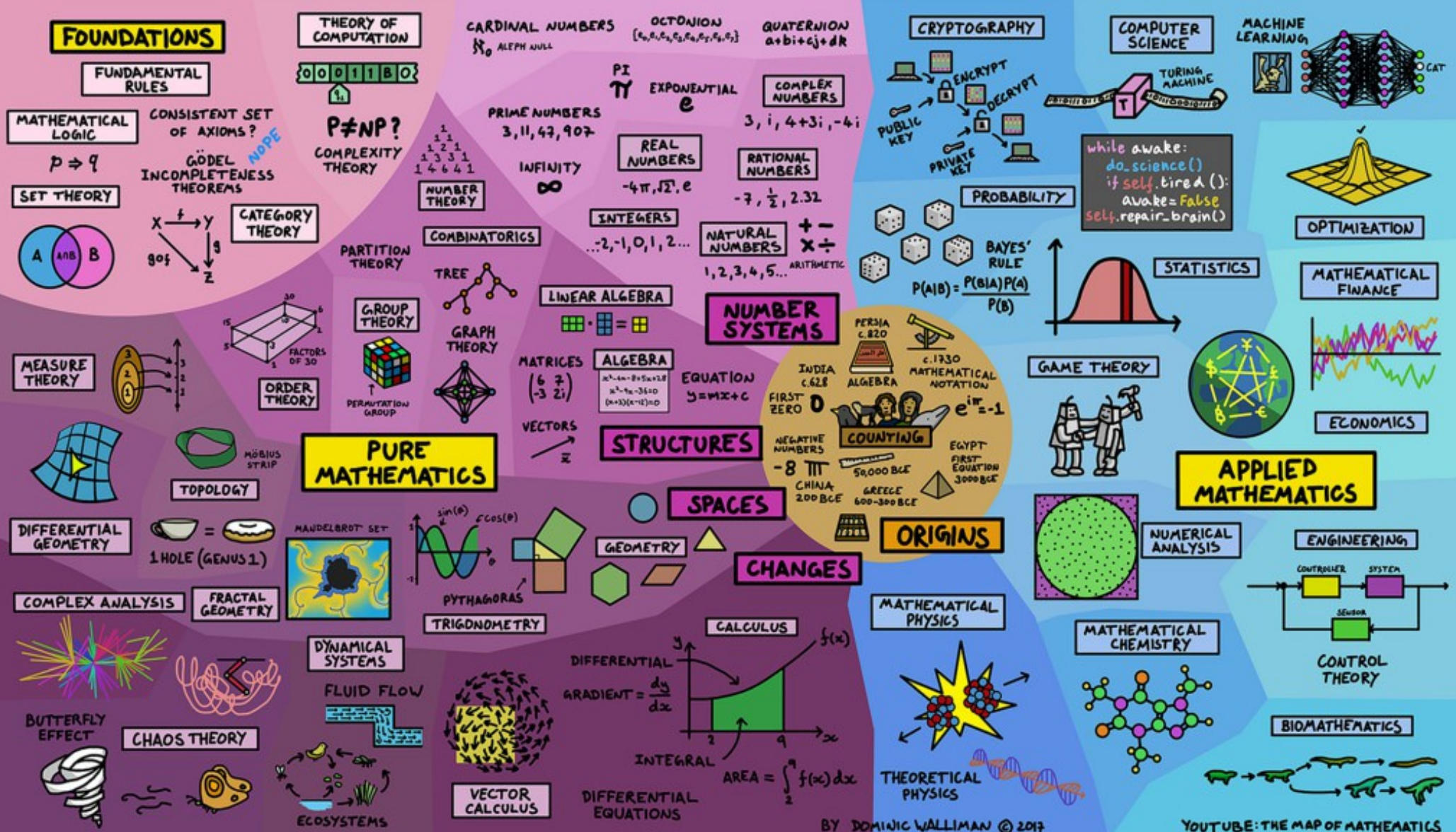


# *Mathematics Publications*

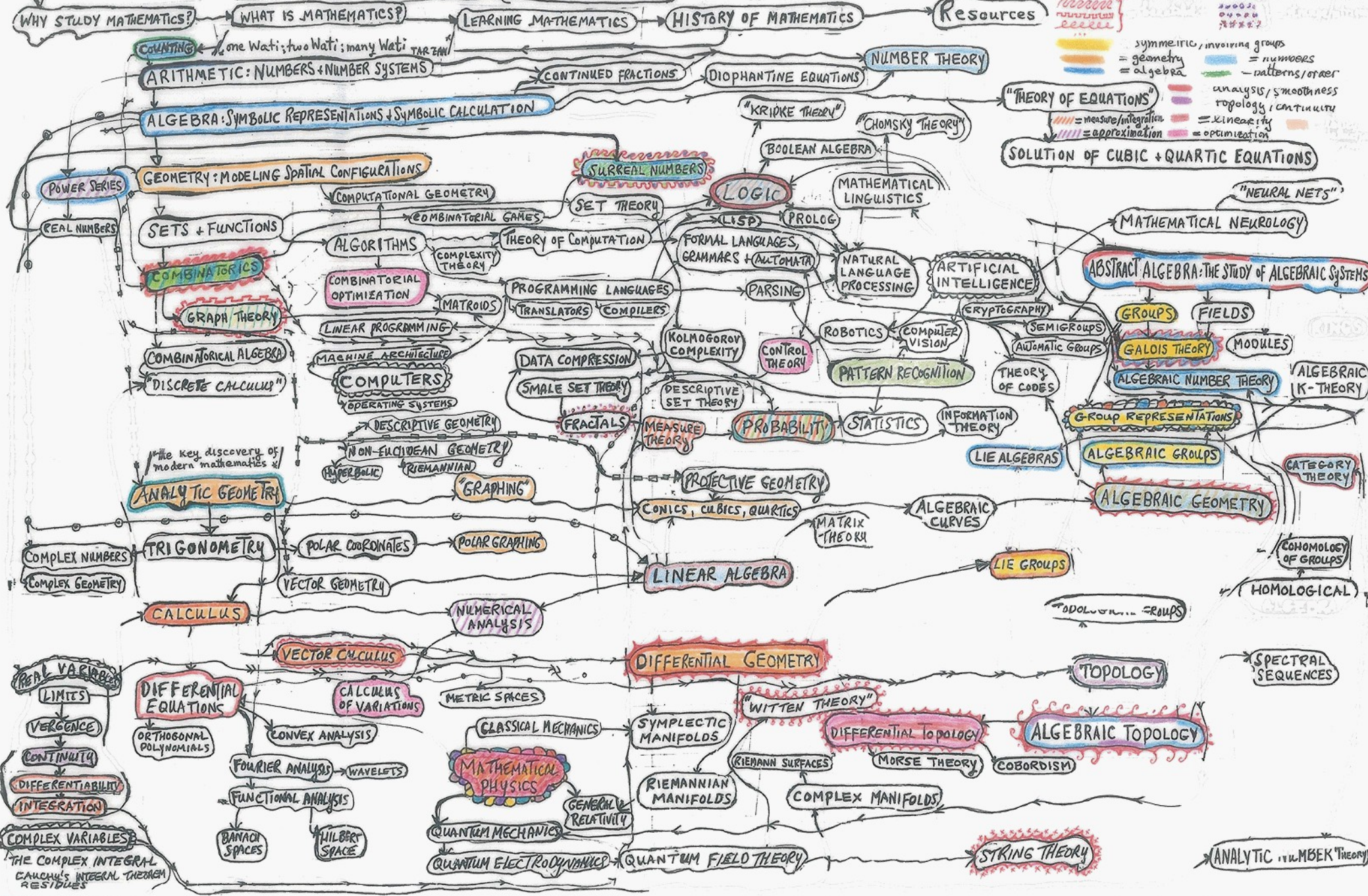


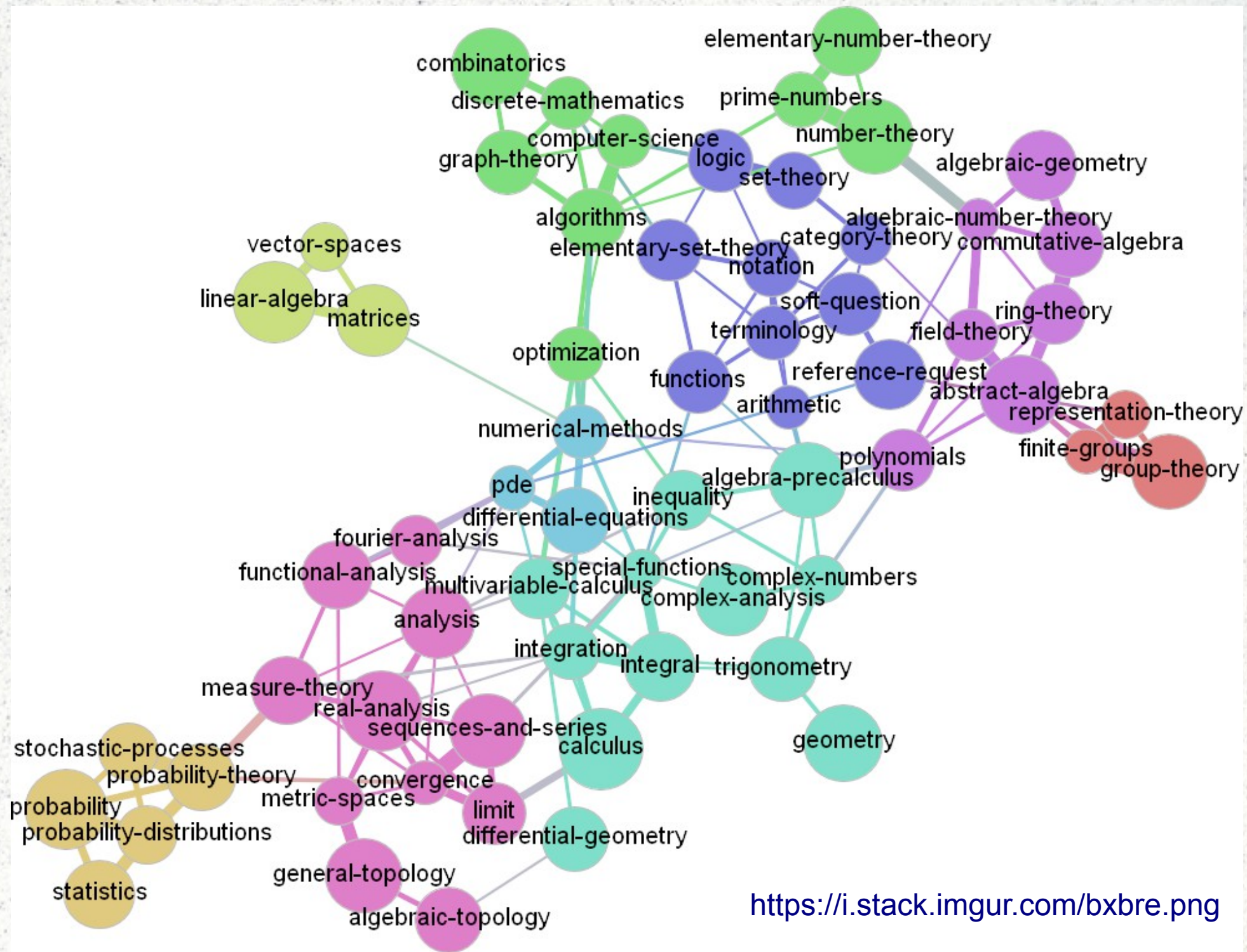
# Mind Maps of Mathematics

<https://www.sciencealert.com/this-mind-boggling-map-explains-how-everything-in-mathematics-is-connected-3>



# (MATHEMATICS — A LEARNING MAP)





<https://i.stack.imgur.com/bxbre.png>

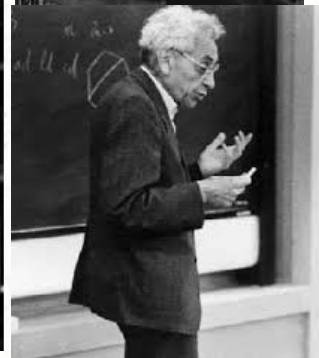
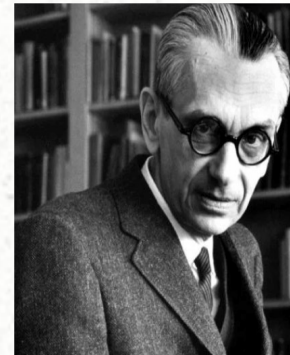
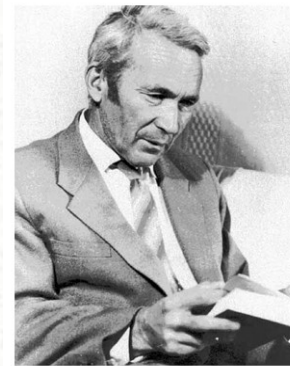


# *Top Numerical Algorithms*

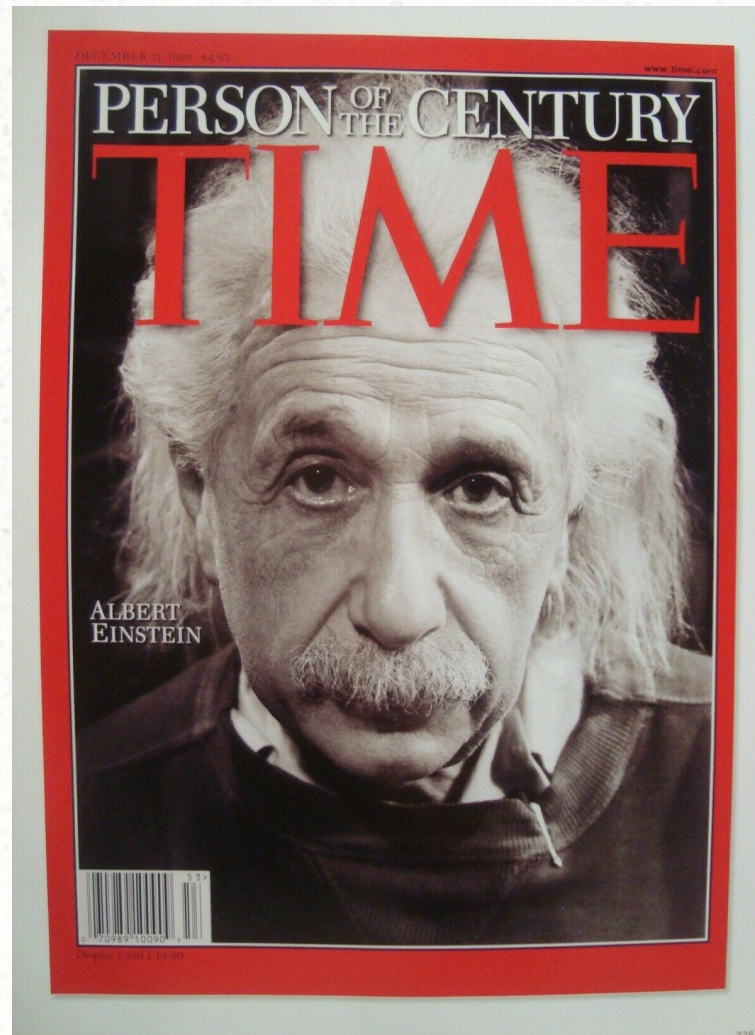
1. Newton and quasi-Newton methods
2. Matrix factorizations (LU, Cholesky, QR)
3. Singular value decomposition, QR and QZ algorithms
4. Monte-Carlo methods
5. Fast Fourier transform
6. Krylov subspace methods (conjugate gradients, Lanczos, GMRES, minres)
7. JPEG
8. PageRank
9. Simplex algorithm
10. Kalman filter

# *Greatest Mathematicians of 20<sup>th</sup> Century?*

- John von Neumann
- Andrey Kolmogorov
- Claude Shannon
- Alexander Grothendieck
- Kurt Godel
- Paul Erdos
- Alan Turing
- Hermann Weyl
- Srinivasa Ramanujan



# *Person of the Century*



*Good Luck on Your Finals!*

