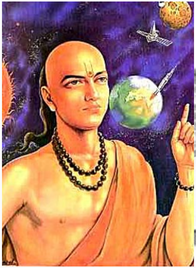
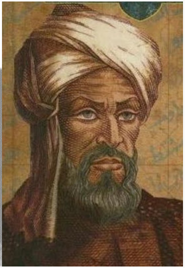
Early Asian Mathematics

Fall 2020 - R. L. Herman





Overview

China

- Unique development
- *Zhoubi Suanjing* - c. 300 BCE
- *Tsinghua Bamboo Slips*, - decimal times table. 305 BCE
- Chinese abacus ($<$190 CE)
- After book burning (212 BCE), Han dynasty (202 BCE–220) produced mathematics works.

India

- *Pingala* (3rd–1st cent. BCE) - binary numeral system, binomial theorem, Fibonacci numbers.
- *Siddhantas*, 4-5 cent. astronomical treatises, trigonometry.

Arabian-Islamic (330-1450)

- Preserved Greek texts
- 7-14th cent. Development of algebra, etc.
- Hindu-Arabic numerals

Chinese Mathematics

- *The Nine Chapters on the Mathematical Art.*
  - 246 word problems on agriculture, business, geometry, engineering, surveying.
  - Proof for the Pythagorean theorem.
  - Formula for Gaussian elimination.
  - Provides values of π. [They had approximated as 3.]
- Computing π
  - Liu Xin (d. 23 AD), $\pi \approx 3.1457$.
  - Zhang Heng (78–139), $\pi \approx 3.1724, 3.162$ using $\sqrt{10}$.
  - Liu Hui (3rd century), commented on the Nine Chapters, $\pi = 3.14159$.
  - Zu Chongzhi (5th century), $\pi = 3.141592$ remained the most accurate value almost 1000 years.
- Zu Chongzhi gave a method (Cavalieri's principle) for volume of a sphere.

Pascal's Triangle

Known by early Chinese mathematicians:

- Systems of linear equations
- Chinese Remainder Theorem
- Square roots
- Pythagorean Theorem
- Euclidean algorithm
- Pascal's Triangle

$$
\begin{aligned}
(a+b)^0 &= 1\\
(a+b)^1 &= a+b\\
(a+b)^2 &= a^2+2ab+b^2\\
(a+b)^3 &= \\
(a+b)^4 &=
\end{aligned}
$$

Figure 1: Binomial Expansions, $(a+b)^n \qquad n = 0, 1, \ldots.$

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- Typical term, $a^{n-k}b^k$, $k = 0, 1, \ldots, n$.

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Figure 1: Binomial Expansions,
$(a+b)^n = \sum_{k=0}^{n} C_{n,k}a^{n-k}b^k, \ n = 0, 1, \ldots.$

Pascal's Triangle

Known by early Chinese mathematicians:

- Systems of linear equations
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- Typical term, $a^{n-k}b^k$, $k = 0, 1, \dots, n$.
- What is the coefficient?

$$
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Pascal's Triangle

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}$$

Figure 2: Pascal's Triangle, $C_{n,k} = \frac{n!}{(n-k)!k!}$

Pascal's Triangle

$$\begin{array}{c} 1 \\ 1 + 1 \\ 1 + 2 + 1 \\ 1 + 3 + 3 + 1 \\ 1 + 4 + 6 + 4 + 1 \\ 1 + 5 + 10 + 10 + 5 + 1 \end{array}$$

Sum each row:

Figure 2: Pascal's Triangle

Pascal's Triangle

$$\begin{array}{ccccccccccccl} & & & & & & 1 & & & & & & = 1 \\ & & & & & 1 & + & 1 & & & & & = 2 \\ & & & & 1 & + & 2 & + & 1 & & & & = 4 \\ & & & 1 & + & 3 & + & 3 & + & 1 & & & = 8 \\ & & 1 & + & 4 & + & 6 & + & 4 & + & 1 & & = 16 \\ 1 & + & 5 & + & 10 & + & 10 & + & 5 & + & 1 & & = 32 \end{array}$$

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Pascal's Triangle

$$
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 & & & & & 1 & + & 1 & & & & & = 2 \\
 & & & & 1 & + & 2 & + & 1 & & & & = 4 \\
 & & & 1 & + & 3 & + & 3 & + & 1 & & & = 8 \\
 & & 1 & + & 4 & + & 6 & + & 4 & + & 1 & & = 16 \\
1 & + & 5 & + & 10 & + & 10 & + & 5 & + & 1 & & = 32
\end{array}
$$

Sum each row:

Sum $= 2^n$.

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Euclidean Algorithm Example

Example
One can only go 23 units or 79 units left or right. How many of each would it take to go from 0 to 1?

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$$79 = 3 \cdot 23 + 10$$

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In Reverse:

$$1 = 10 - 3 \cdot 3$$

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$$\begin{aligned} 1 &= 10 - 3 \cdot 3 \\ &= 10 - 3(23 - 2 \cdot 10) \end{aligned}$$

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In Reverse:

$$\begin{aligned} 1 &= 10 - 3 \cdot 3 \\ &= 10 - 3(23 - 2 \cdot 10) \\ &= 7 \cdot 10 - 3 \cdot 23 \\ &= 7 \cdot (79 - 3 \cdot 23) - 3 \cdot 23 \\ &= 7 \cdot 79 - 24 \cdot 23 \end{aligned}$$

Euclidean Algorithm Example

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One can only go 23 units or 79 units left or right. How many of each would it take to go from 0 to 1?

$$\bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \underset{0}{\bullet} \quad \underset{1}{\bullet} \quad \bullet \quad \bullet \quad \bullet \quad \bullet$$

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Thus, $m = 7, n = -24.$

Chinese Remainder Theorem

The Chinese remainder theorem: If one knows the remainders of the Euclidean division of an integer x by several integers, then one can determine uniquely the remainder of the division of x by the product of these integers, assuming the divisors are pairwise coprime. Earliest - Sun-tzu in *Sun-tzu Suan-ching*.

If $p_1, p_2, \dots, p_n$ are relatively prime, then

$$\begin{aligned} x &= r_1 \pmod{p_1} \\ x &= r_2 \pmod{p_2} \\ &\vdots \\ x &= r_n \pmod{p_n} \end{aligned}$$

always has a solution.

Chinese Remainder Theorem Example

Example

$$\begin{aligned} x &= 2 \pmod 3 \\ x &= 3 \pmod 5 \\ x &= 2 \pmod 7 \end{aligned}$$

First equation means $x = 3n + 2$. Insert into second:

$$\begin{aligned} 3n+2 &= 3 \pmod 5 \\ 3n &= 1 \pmod 5 \\ 3n &= 6 \pmod 5 \\ n &= 2 \pmod 5 \end{aligned}$$

So,

$$\begin{aligned} x &= 3n+2 \\ &= 3(5m+2) \\ &= 15m+8. \end{aligned}$$

From third equation

$$\begin{aligned} 15m+8 &= 2 \pmod 7 \\ 15m+1 &= 2 \pmod 7 \\ 15m &= 1 \pmod 7 \\ 15m &= 15 \pmod 7 \\ m &= 1 \pmod 7 \end{aligned}$$

Therefore, $m = 7k + 1$ and $x = 105k + 23$.

Indian Mathematics (500-1200)

- Major mathematicians
  - Aryabhata (476-550?)
  - Bhaskara I (600-680)
  - Brahmagupta (598-668)
  - Bhaskara II (1114-1185)
  - Madhava (1350-1425)
- Contributions
  - Algebra
  - Geometry
  - Trigonometry
  - Spherical trigonometry
  - Diophantine Equations
  - Mathematical astronomy
  - Place-value decimal system

Brahmagupta:
$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$
$s = \frac{1}{2}(a+b+c+d)$ is semiperimeter

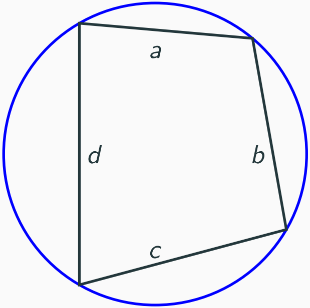


Figure 3: Cyclic Quadrilaterals

Aryabhata (476-550)

- Major work, *Aryabhatiya*, mathematics and astronomy,

  arithmetic, algebra,
  plane trigonometry,
  spherical trigonometry.
  continued fractions,
  quadratic equations,
  sums of power series, and
  table of sines.
- 108 verses, 13 introductory verses
- Relativity of motion
- *Arya-siddhanta*,

  Astronomical computations
  Astronomical instruments

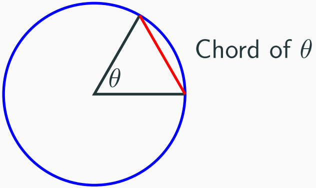
Figure 4: Aryabhata on the grounds of IUCAA, Pune.

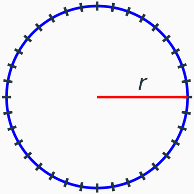
Table of Sines

- Introduction of sine
- Aryabhata's sine table
- Based on half chords
  vs Hipparchus, Menlaus, Ptolemy.
- Also, provided differences
- From Babylonians, base 60
  degrees, minutes, seconds
- Circumference $= 21600'$.
- Aryabhata, $\pi = 3.1416$
- Bhaskara I approximation

$$\sin\theta \approx \frac{4\theta(180-\theta)}{40500-\theta(180-\theta)}.$$

- Mādhava's - more accurate.





$$360^o = 21600'$$

$$r = \frac{21600}{3.1416} \approx 3438$$

Pell's Equation, $x^2 - Ny^2 = 1$, N Nonsquare

Brahmagupta:

$$(x_1^2 - Ny_1^2)(x_2^2 - Ny_2^2) = (x_1x_2 + Ny_1y_2)^2 - N(x_1y_2 + x_2y_1)^2$$

If $x_1^2 - Ny_1^2 = k_1$ and $x_2^2 - Ny_2^2 = k_2$, then

$$\begin{aligned} x &= x_1x_2 + Ny_1y_2 \\ y &= x_1y_2 + x_2y_1 \end{aligned}$$

solves $x^2 - Ny^2 = k_1k_2$.

This gives a composition of triples, (x_1, y_1, k_1) and (x_2, y_2, k_2) to give (x, y, k_1k_2).

Example (Brahmagupta) $x^2 - 92y^2 = 1$.

Note: $10^2 - 92(1)^2 = 8$. Thus, triple $= (10, 1, 8)$.

Pell's Equation (cont'd)

- $10^2 - 92(1)^2 = 8 \to (10, 1, 8)$.
- Compose $(10, 1, 8)$ with itself. $(10 \cdot 10 + 92 \cdot 1 \cdot 1, 10 \cdot 1 + 1 \cdot 10, 8 \cdot 8) = (192, 20, 64)$
- or, $192^2 - 92(20)^2 = 64$
  $24^2 - 92\left(\frac{5}{2}\right) = 1$
- Compose $(24, \frac{5}{2}, 1)$ with itself: $(1151, 120, 1)$.
- Bhaskara II (1150) - cyclic process always works.

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- Proved by Lagrange (1768) $gcd(a, b) = 1,\ a^2 - Nb^2 = k$.
- Compose (a, b, k) with $(m, 1, m^2 - N)$ gives $(am + Nb, a + bm, k(m^2 - N))$.
- Rescale

$$\left(\frac{am + Nb}{k}, \frac{a + bm}{k}, m^2 - N\right)$$

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- Fermat (1657), $x^2 - 61y^2 = 1$, $x = 1766319049$, $y = 226153980$.

Number Systems

- Egyptian - hieroglyphics
- Greek - letters
- Roman - I, V, X, L, C, D, M, I, III, IV, ...
- Chinese - counting boards
- Hindu

  $\sim$ 600: 1,2,...,9
  800: placed digits ,powers of 10, zero.
- Arabic - Adopted Hindu for Hindu–Arabic numeral system.

Arabian-Islamic Mathematics

- Absorbed Greek and Hindu math
- Contributed to

  Algebra, geometry, astronomy,
  plane and spherical trigonometry.
- Al'Khwarizmi (780-850)
  - al Jabr and algorithm
  Solved equations with variables.
- Omar Khyayyam (1048-1131) studied cubic equations, and the intersection of conics, Khyayyam's triangle, parallel postulate.
- al-Kashi (1380-1429), decimal and sexagesimal fractions, good approximation to π. Law of Cosines.

Rise of European Mathematics

- Fall of the Roman Empire
- Middle Ages, Medieval Period, 5th to the 15th century.
- Byzantine Empire (330-1453) - Church split, preservation of Greek works.
- Al'Khwarizmi's work and Euclid translated.
- Crusades (1095-1291), The Plague (1347 to 1351), Church - monasteries
- Al'Khwarizmi's work and Euclid translated.
- Johannes Gutenberg' printing press, 1440.
- Renaissance (1400-1600) and the Age of Discovery.
- Questioning of Aristotle
- Church of England (1534), Protestant vs Roman Catholic

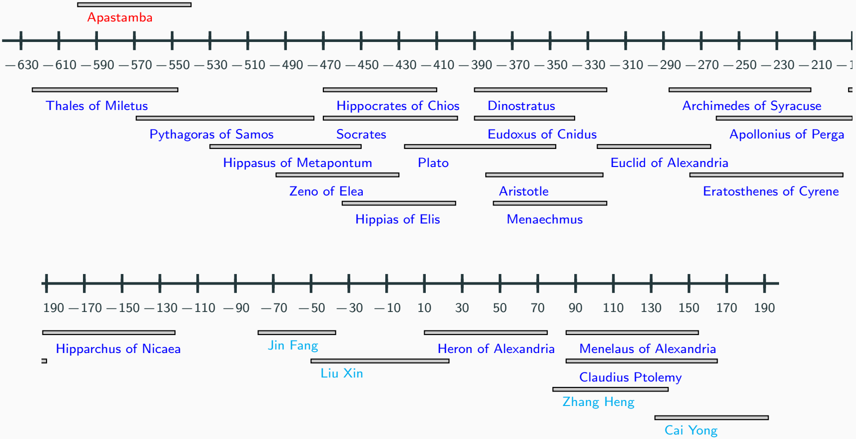
Medieval Mathematicians

- Leonardo of Pisa (Fibonacci) (1200).
- Nicole Oresme (1323-1382), coordinate geometry, fractional exponents, infinite series.
- Johann Müller Regiomontatus (1436-1476), separated trigonometry from astronomy.
- And others:

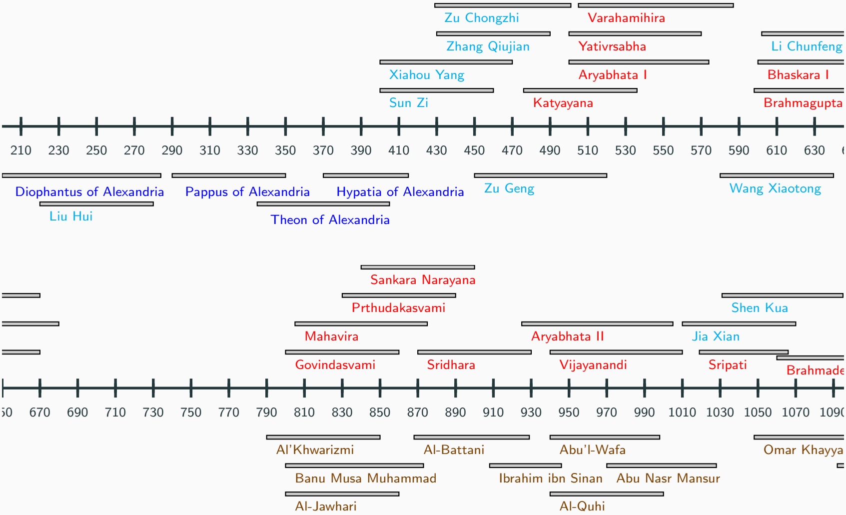
Roger Bacon (1214-1292)	William of Ockham (1288-1348)
Filippo Brunelleschi (1377-1446)	Leone Battista Alberti (1404-1472)
Nicholas of Cusa (1401-1464)	Luca Pacioli (1445-1517)
Leonardo da Vinci (1452-1519)	Scipione del Ferro (1465-1526)
Nicolaus Copernicus (1473-1543)	

- Rise of European Mathematics .. beginning in Italy.

Timeline of Ancient Mathematicians i



Timeline of Ancient Mathematicians ii



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