General Relativity – Part I

I. Geometry

a. Sphere

i. Sum of Interior Angles of a Triangle = $p + \frac{A}{a^2}$

ii. $\frac{C}{r} = 2p\,\frac{\sin(r/a)}{r/a}$

b. Line Elements

i. $dS^2 = dx^2 + dy^2$, $dS^2 = dr^2 + (rdf)^2$, $dS^2 = a^2(dq^2 + \sin^2 q\,df^2)$

ii. $dS^2 = a^2(dq^2 + f^2(q)df^2)$

iii. $C(q) = \int_0^{2p} af(q)\,dq$, $d_{pole-pole} = \int_0^{p} a\,dq$

II. Space, Time and Gravity

a. Coordinate Transformations – displacement(translation), rotation, uniform motion, Galilean transformation

b. Rotation of Axes $x' = x\cos j + y\sin j$, $y' - -x\sin j + y\cos j$, $z' = z$

c. Principle of Relativity

d. Newtonian Gravity

i. $\nabla^2\Phi = -4pGm$, $F = -m\nabla\Phi$

e. Variational Principle: $S[x(t)] = \int_{t_A}^{t_B} L(\dot{x}(t), x(t))\,dt$ $dS = 0 \Rightarrow -\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) + \frac{\partial L}{\partial x} = 0$

III. Special Relativity

a. Einstein's Postulates

b. Simultaneity

c. Time Dilation $dt = dt\sqrt{1 - V^2/c^2}$ or $\Delta t' = g\Delta t_0$, $g = (1 - v^2/c^2)^{-1/2}$

d. Length Contraction $L' = L_0/g$

e. Line element $ds^2 = -(cdt)^2 + dx^2 + dy^2 + dz^2$

f. Spacetime Diagrams and Light Cones

g. Lorentz Transformations/Boosts

$t' = g(t - vx/c^2), x' = g(x - vt), y' = y, z' = z$

hyperbolic angle $v = c\tanh q$

h. Proper time $dt^2 = -ds^2/c^2$

i. Addition of Velocities $V^{x'} = \frac{dx'}{dt'} = \frac{V^x - v}{1 - vV^x/c^2}$, etc.

IV. Four Vectors and Dynamics

a. $\mathbf{a} = a_a\mathbf{e}^a$ for $a = 0,1,2,3$ - using Einstein Summation Convention

b. $\mathbf{a}\cdot\mathbf{b} = h_{ab}a^a b^b$ for $h = diag(-1,1,1,1)$

c. $ds^2 = h_{ab}dx^a dx^b$

d. $x^a = x^a(t), u^a = \frac{dx^a}{dt} = (g, g\vec{V})$, $\mathbf{u}\cdot\mathbf{u} = -1$

e. $m\mathbf{a} = m\frac{d\mathbf{u}}{dt} = \mathbf{f}$, $\mathbf{f}\cdot\mathbf{u} = 0$

f. $\mathbf{p} = m\mathbf{u} \Rightarrow \mathbf{p}\cdot\mathbf{p} = -m^2$, $p^a = (E, \vec{p}) = (mg, mg\vec{V})$ and finally: $L = \sqrt{-h_{ab}\frac{dx^a}{ds}\frac{dx^b}{ds}}$