Spring Dashpot Model

A 1000 kg mass is dropped from 10.0 m onto a massless plank supported with a spring-dashpot system. (The dashpot provides damping). The spring has spring constant k and the dashpot characterized by damping constant b. It is desired that the mass drops on to the plank resulting in the system settling at a new equilibrium 0.20 m lower than the initial position as quickly as possible without overshooting this equilibrium. a) Find the spring constant. b) Find the damping constant. c) Find to two significant digits the time required for the platform to settle within 1.00 mm of the final position.

```
> restart: with(plots):
Warning, the name changecoords has been redefined
```

Insert Constants

```
> g:=9.8: m:=1000: x1:=0.2: h:=10:
```

Determine k value

```
> k:=2*m*g*h/x1^2;
```

$$k := 0.4900000000\ 10^{7}$$

Find $\beta = \sqrt{\frac{k}{m}}$ and $b = 2\, m\, \beta$ based on critical damping

```
> omega:=sqrt(k/m): beta:=omega; b:=2*m*beta;
```

$$\beta := 70.00000000$$

$$b := 140000.0000$$

Initial conditions from time spring is to be compressed

```
> x0:=x1: v0:=-sqrt(2*g*h):
```

Solve ODE for position as function of time

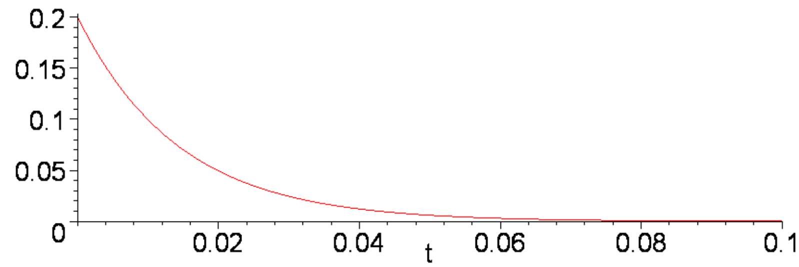
```
> sol:=dsolve({diff(x(t),t$2)+2*omega*diff(x(t),t)+omega^2*x(t),x(0)
  =x0,D(x)(0)=v0}):
```

Convert solution from previous result to usable form

```
> f:=unapply(rhs(sol),t):
```

Plot solution and solve for time t when x=0.001 (= 1 mm)

```
> plot(f(t),t=0..0.1); fsolve(f(t)=0.001);
```



$$0.07569024809$$

Back of envelope computation - since t is small, only need to solve $x(t) = c_1\, \mathbf{e}^{(-\beta\, t)}$. Thus, $t = \frac{\ln\left(\frac{c_1}{x}\right)}{\beta}$

```
> fsolve(0.2*exp(-omega*t)=0.001); ln(x0/.001)/beta;
```

$$0.07569024809$$