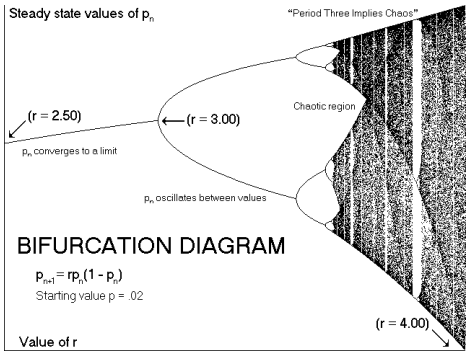
MAT 475 & MAT 564 – Fall 2006

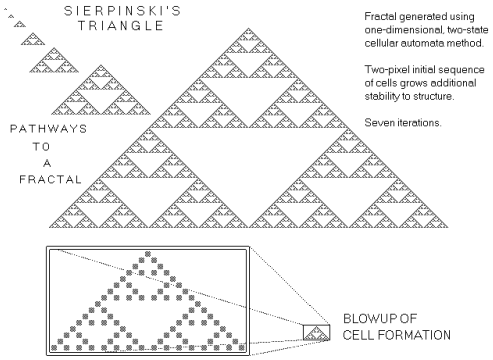
Dynamical Systems and Chaos

Dr. R. Herman

Prerequisites: Linear algebra and an interest in learning about new discoveries in nonlinear mathematics; exposure to differential equations is recommended.

Nonlinear Dynamical Systems and Chaos. Study of nonlinear dynamical systems, leading to chaotic behavior. Topics are to include characteristics of nonlinear systems; iteration of maps; universality theory; continuous time systems and difference equations; stability analysis, fixed points, bifurcations, dissipative flows and attractors; measures of chaos; selected advanced topics, one-dimensional Lyapunov exponents, Poincare-Bendixson theorem, quasiperiodicity, turbulence, controlling chaos, Hamiltonian systems, KAM tori.

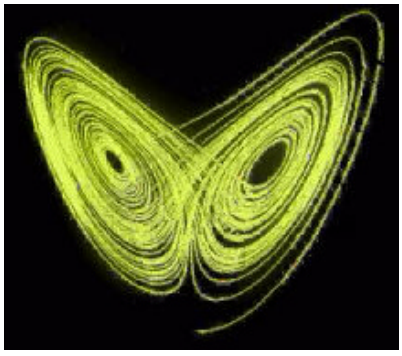


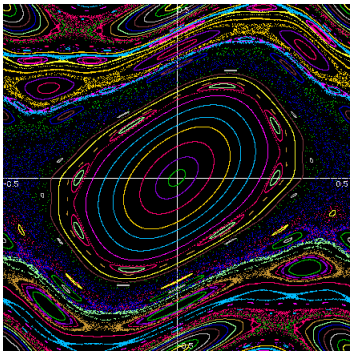


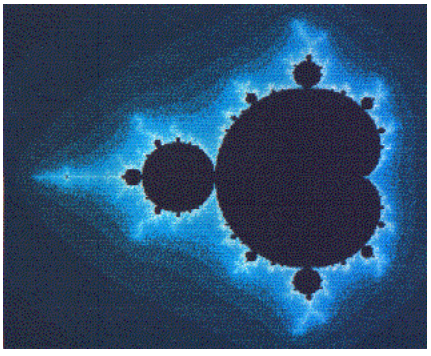
What is Chaos?

Many simple nonlinear deterministic systems can behave in an apparently unpredictable and chaotic manner. Chaos was not fully appreciated until the widespread availability of digital computers for numerical simulations and the demonstration of chaos in various physical systems. The study of chaotic systems has undergone explosive growth in the past two decades. The ideas of chaos have been very fruitful in such diverse disciplines as biology, economics, chemistry, engineering, fluid mechanics, and physics.

A dynamical system is one whose evolution is deterministic: its future motion is determined by its current state and it's past history. A chaotic dynamical system is characterized principally by its sensitivity to initial conditions. A well-known example of this is the weather. Small changes in weather patterns can result in large changes later on.







"If we knew exactly the laws of nature and the situation of the universe at the initial moment, we could predict exactly the situation of that same universe at a succeeding moment. but even if it were the case that the natural laws had no longer any secret for us, we could still only know the initial situation approximately. If that enabled us to predict the succeeding situation with the same approximation, that is all we require, and we should say that the phenomenon had been predicted, that it is governed by laws. But it is not always so; it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon." – Henri Poincaré, 1903

More information about chaos can be found at **http://people.uncw.edu/hermanr/chaos**

***The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living*.- Henri Poincaré.**