Solving Differential Equations with Simulink

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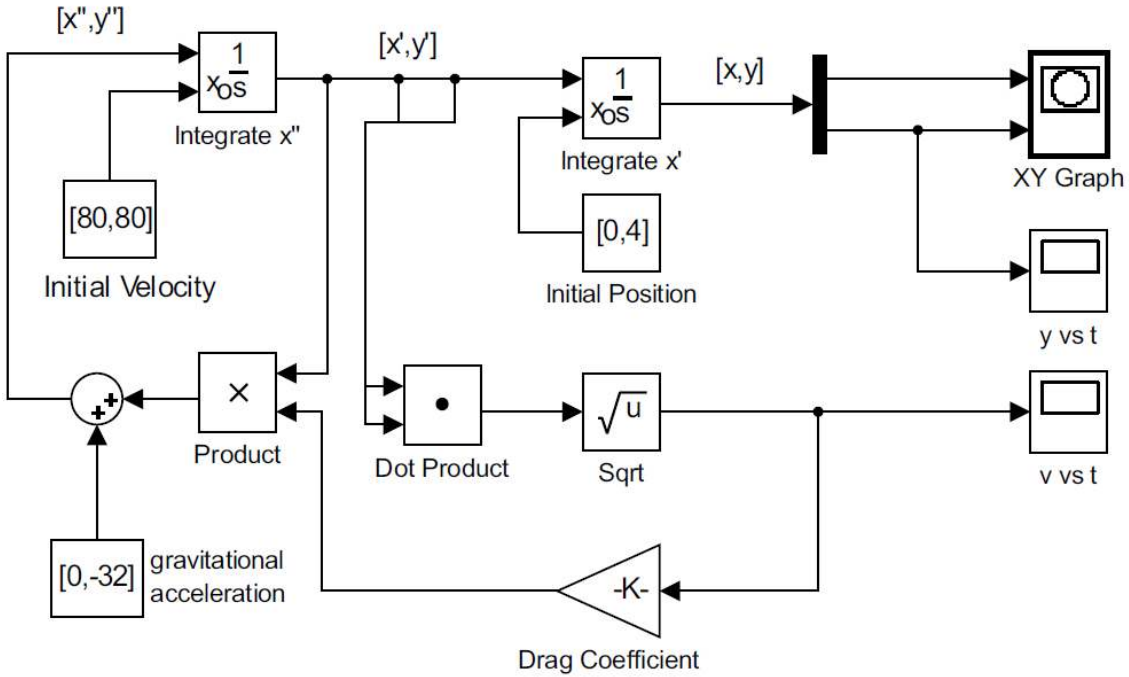
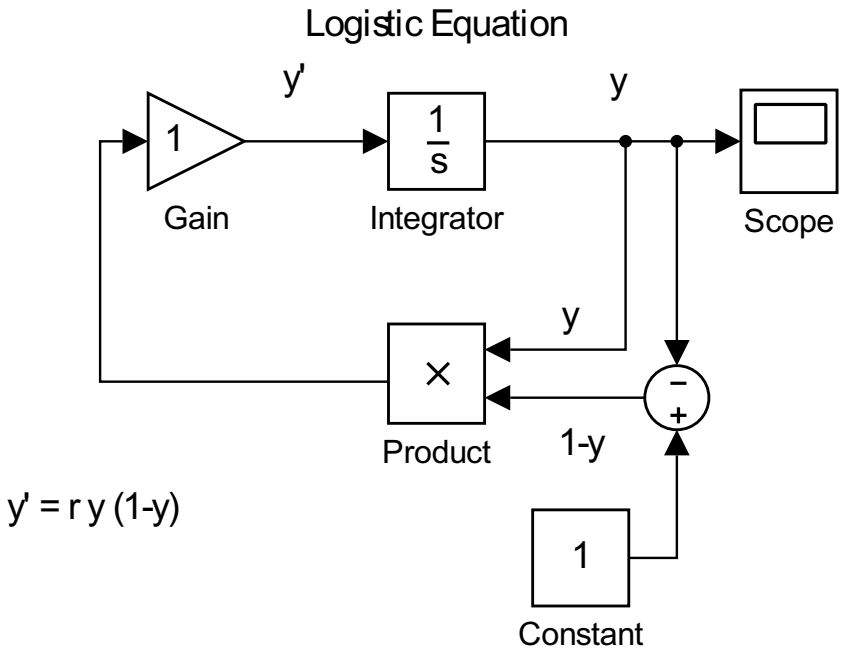


Outline

Simulink

- What is Simulink
  - Graphical environment for designing simulations
  - Product of Mathworks
  - Select and connect blocks
- Use in Differential Equations
  - Project component of class
  - Modeling applications



Solving a Differential Equation

Consider initial value problem:

$$\frac{dx}{dt} = f(x), \quad x(0) = x_0.$$

Solution

$$x(t) = x_0 + \int_0^t f(x(t))\, dt.$$

Think of the solution as

$$x(t) = \int x'(t)\, dt.$$

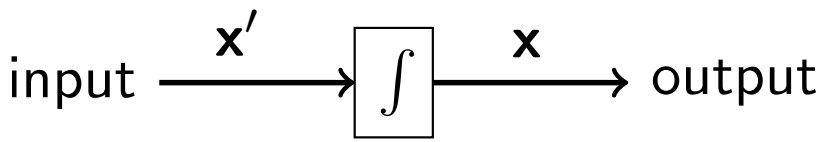


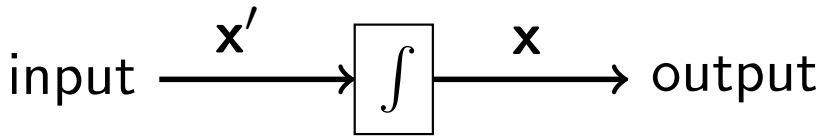
Figure : Schematic for a general system.

Simulink Model of a Differential Equation

Modeling

$$x(t) = \int x'(t)\,dt = \int f(x(t))\,dt.$$

Schematic



Simulink Model

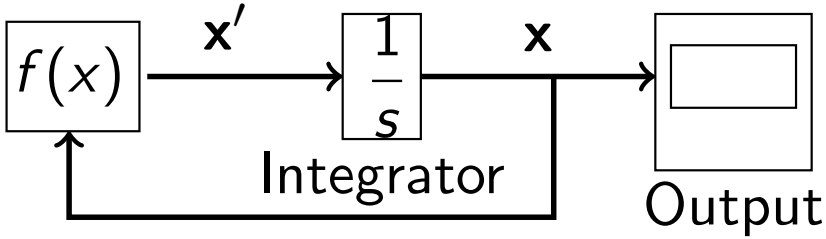
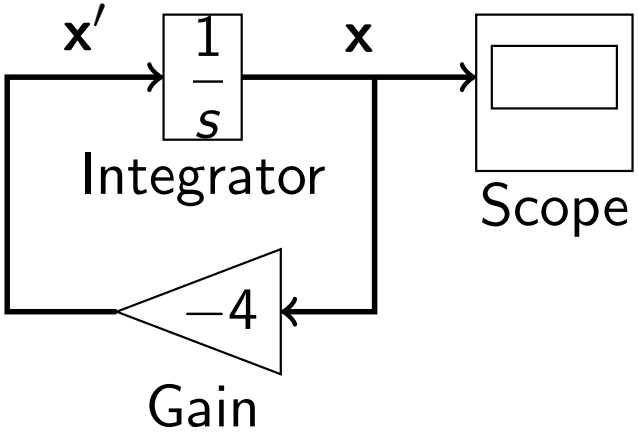
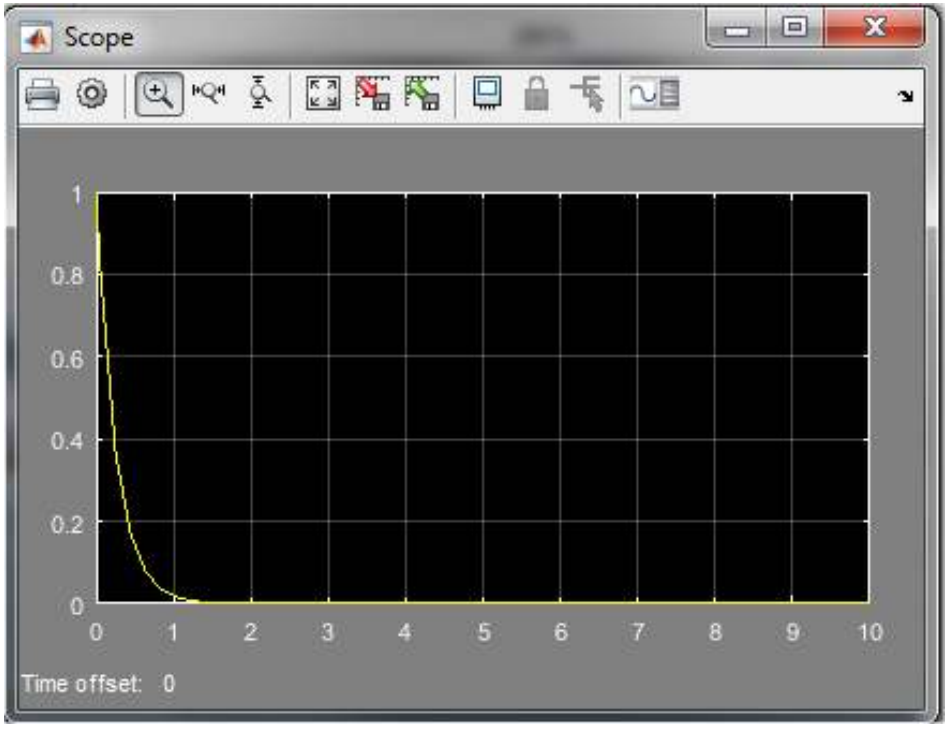


Figure : Model for solving $x' = f(x)$.

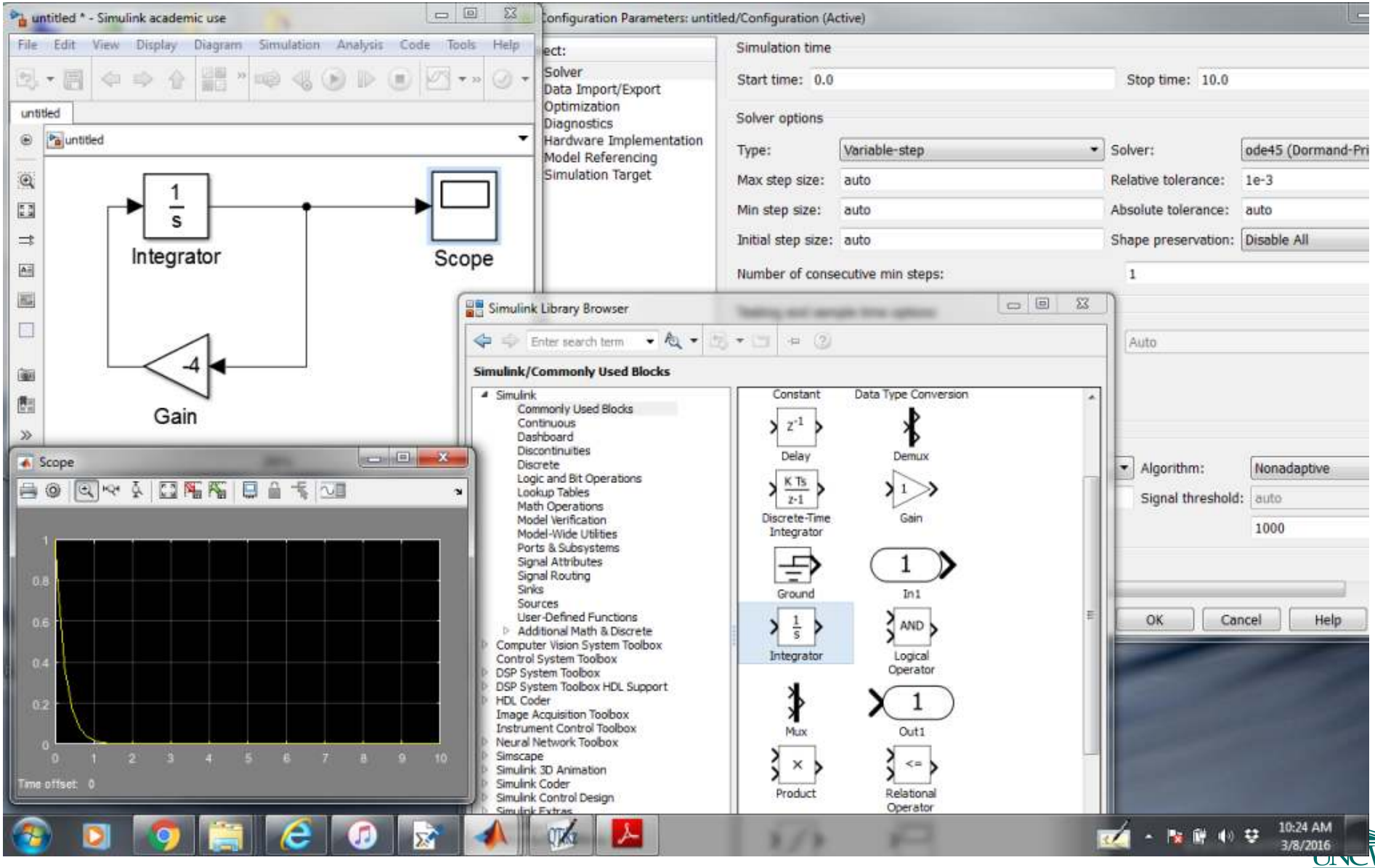
First Order Differential Equation - Example 1

Solve $x' = -4x, \quad x(0) = 1.$

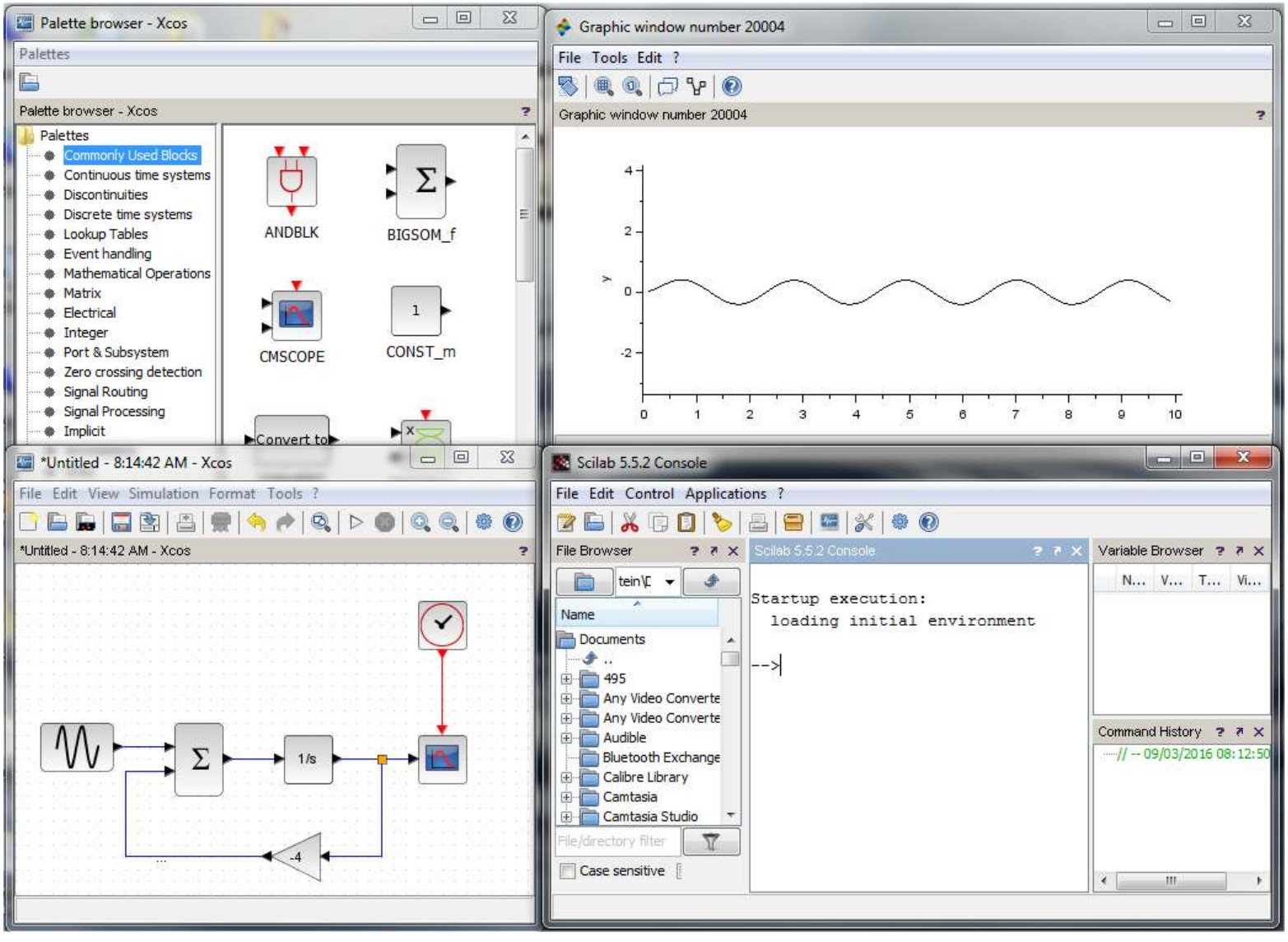




Simulink Workspace

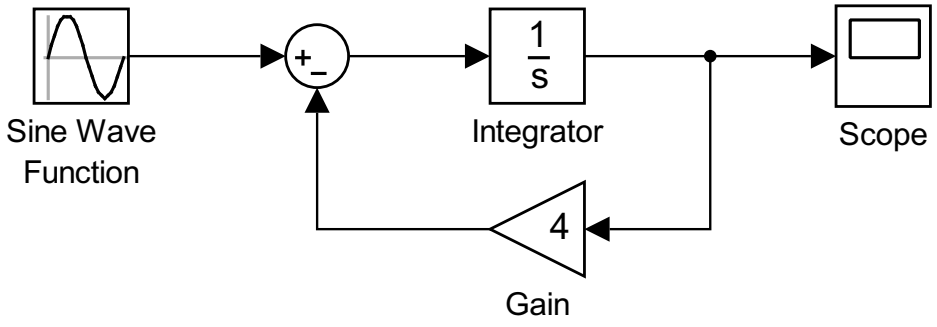


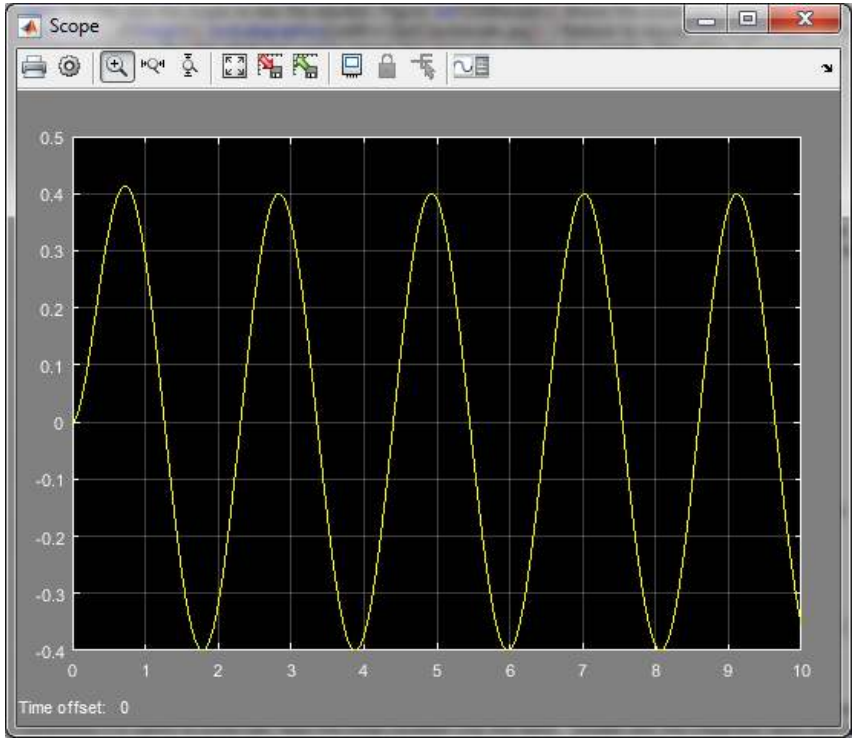
Scilab's Xcos Workspace



First Order Differential Equation - Example 1

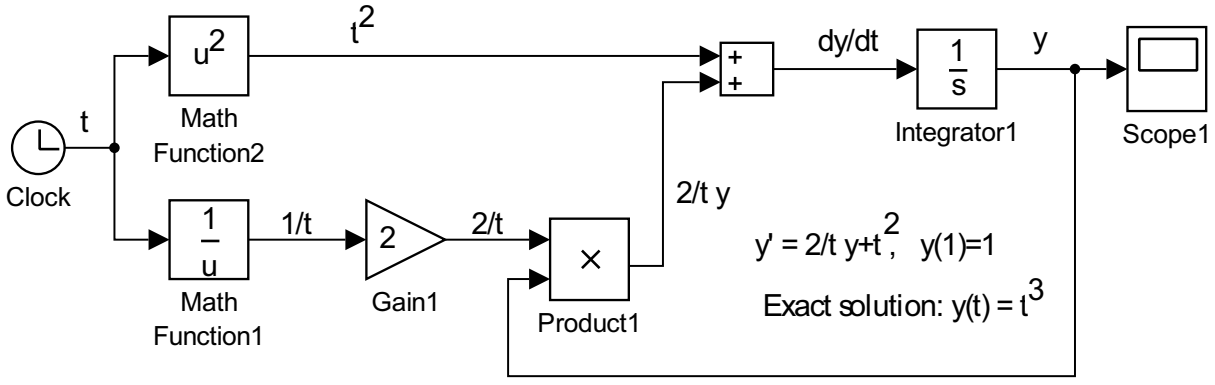
Solve $x' = 2\sin 3t - 4x, \quad x(0) = 0.$

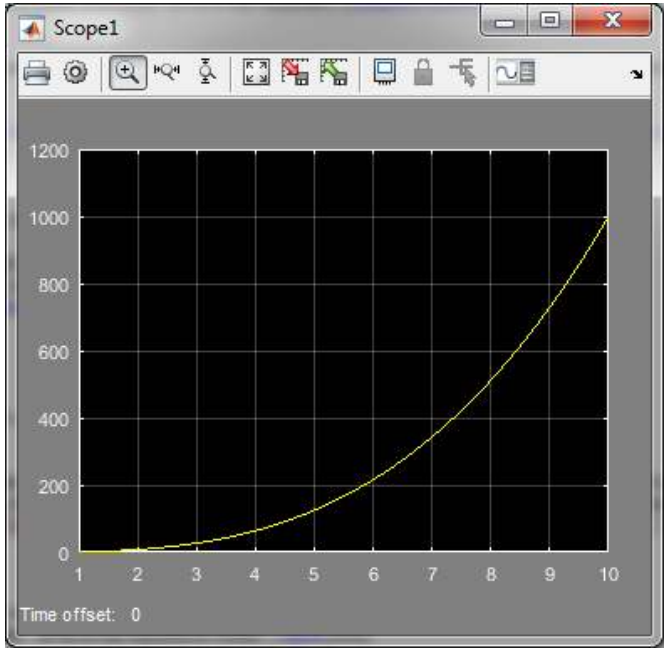




First Order Differential Equation - Example 3

Solve $y' = \frac{2}{t}y + t^2, \quad y(1) = 1.$



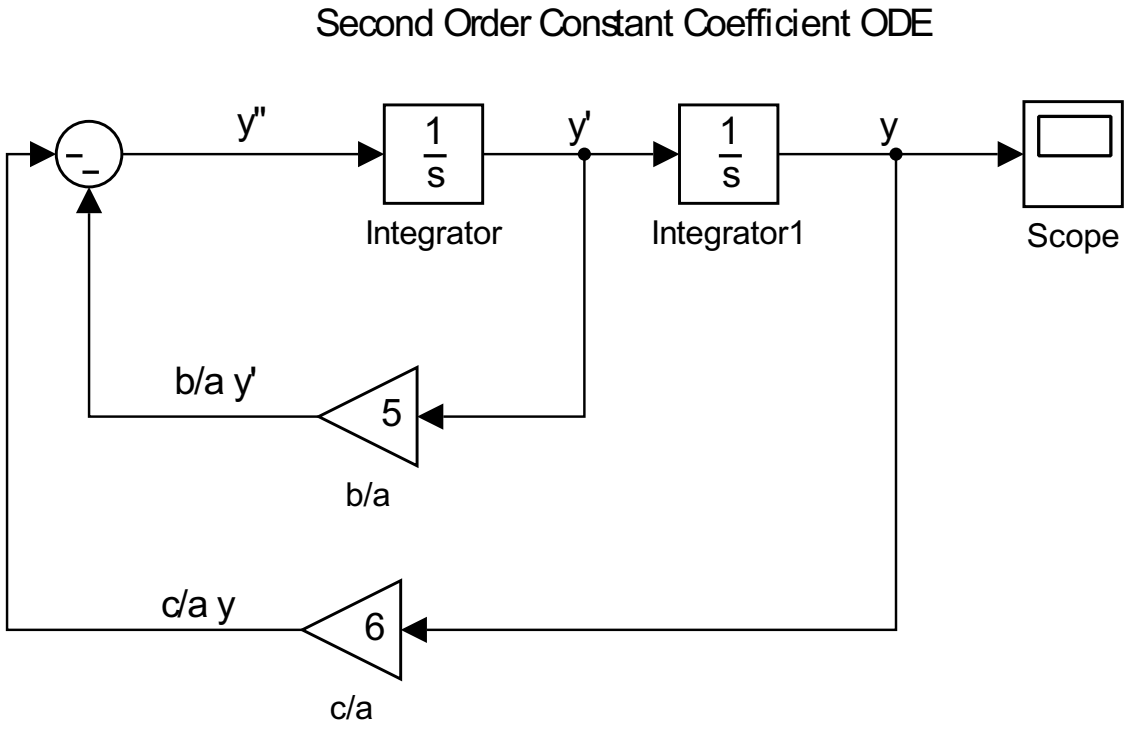


Second Order ODEs

Solve $ay'' + by' + cy = 0, \quad y(0) = y_0, y'(0) = v_0.$

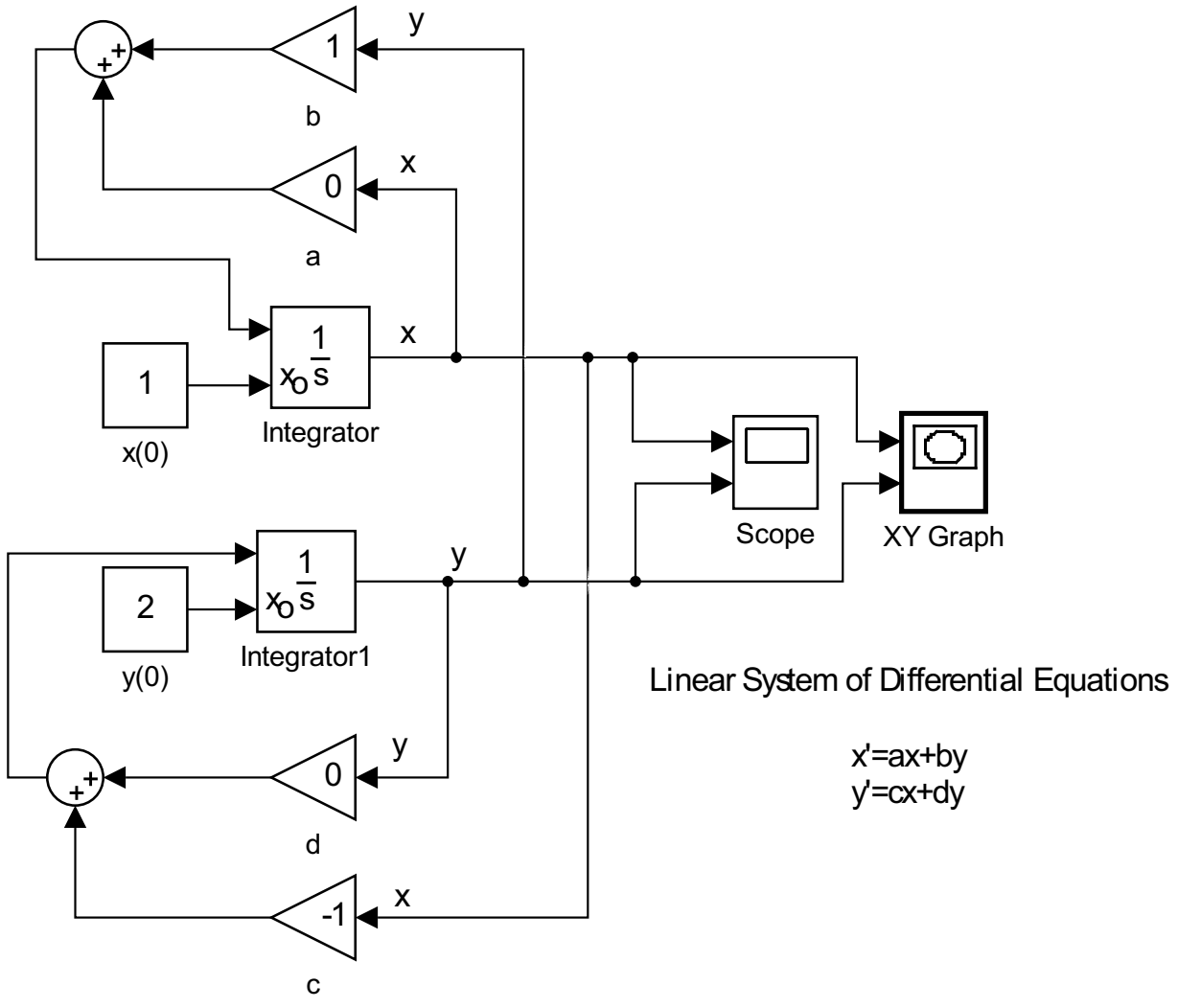
$$y = \int y'\,dx, \quad y' = \int y''\,dx,$$

$$y'' = -\frac{b}{a}y' - \frac{c}{a}y.$$



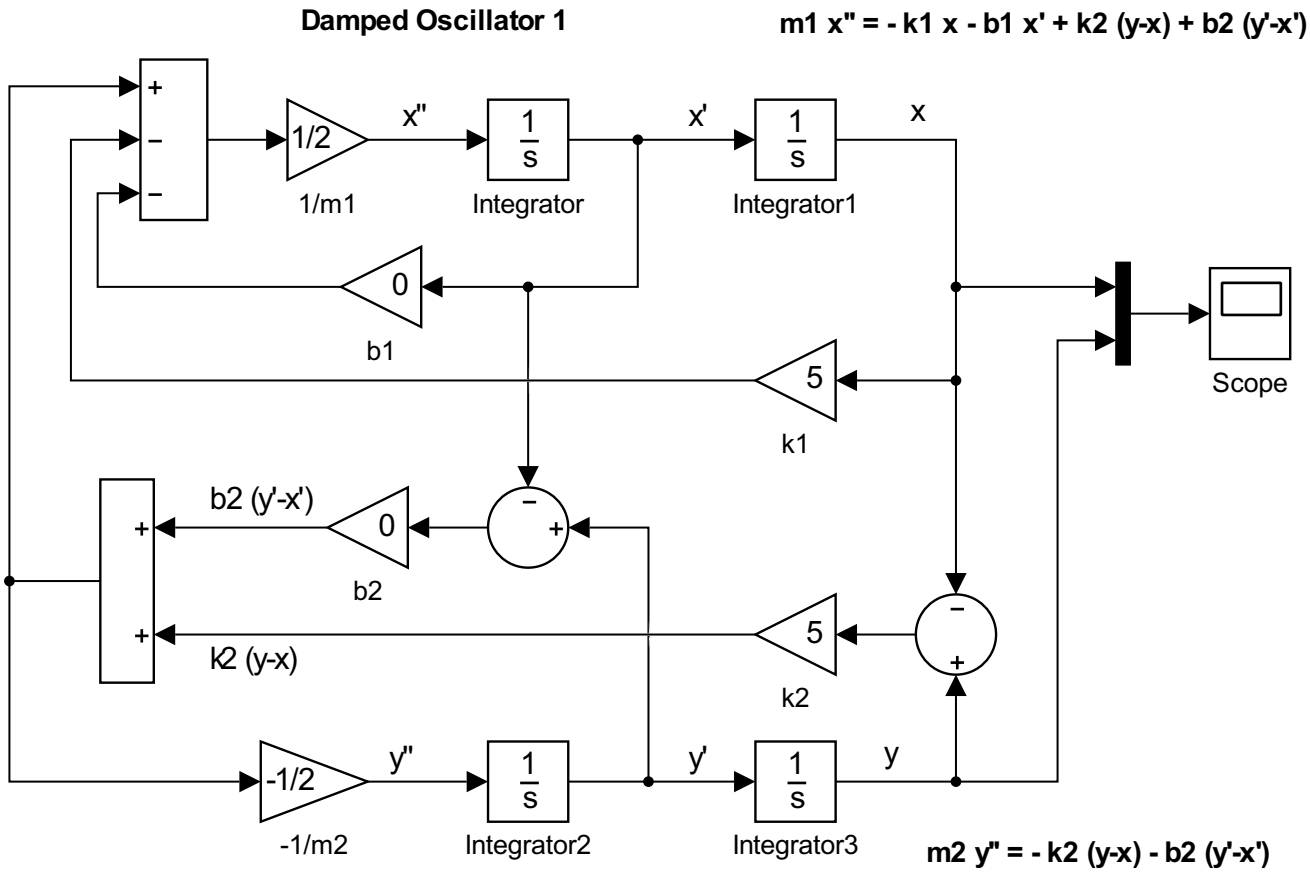
Linear Systems of Differential Equations

$$
\begin{aligned}
x' &= ax + by \\
y' &= cx + dy.
\end{aligned}
$$



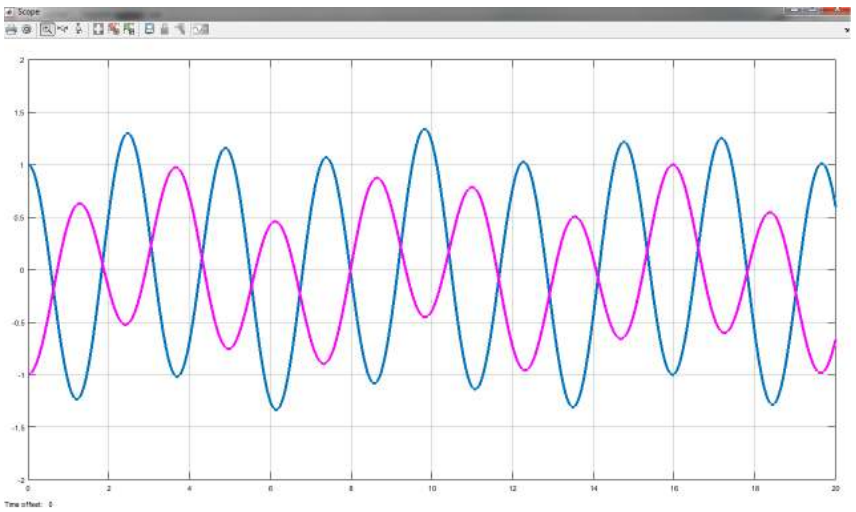
Coupled Oscillators

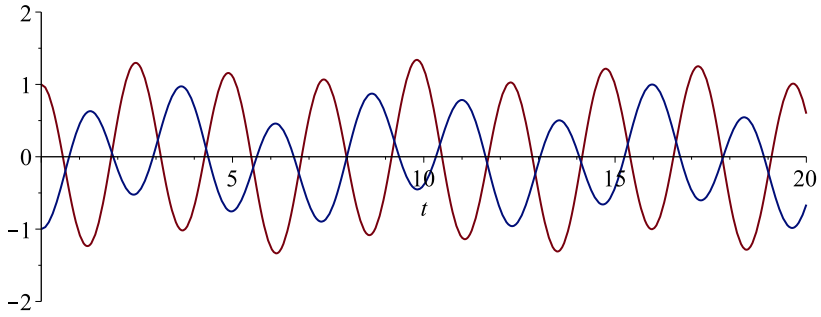
$$
\begin{aligned}
x'' &= -k_1 x - b_1 x' + k_2(y - x) + b_2(y' - x') \\
y'' &= -k_2(y - x) - b_2(y' - x').
\end{aligned}
$$



Coupled Oscillators - Solutions

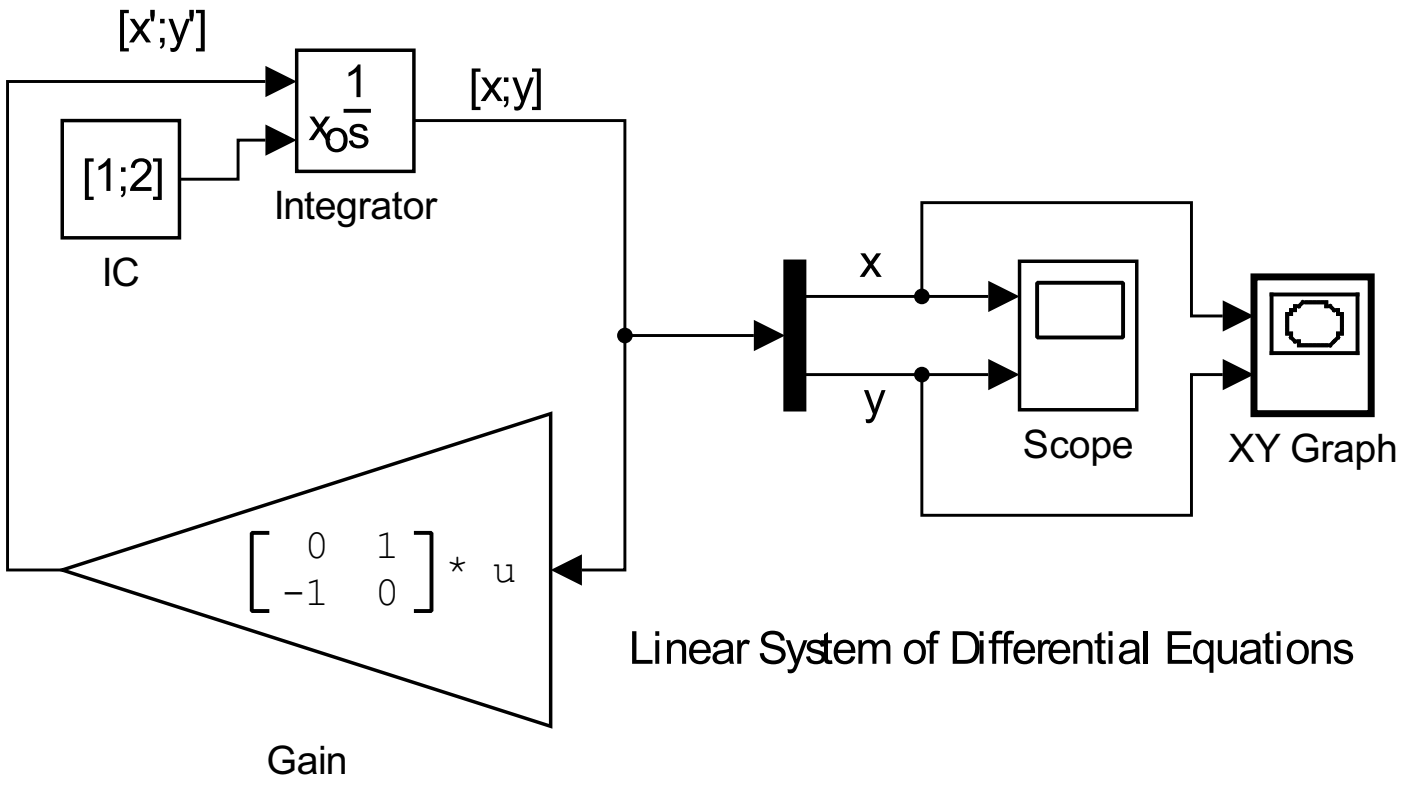
$$
\begin{aligned}
x(t) &= \cos\left(\frac{5}{4}\sqrt{2}t\right)\cos\left(\frac{1}{4}\sqrt{10}t\right) - \frac{3}{5}\sqrt{5}\sin\left(\frac{5}{4}\sqrt{2}t\right)\sin\left(\frac{1}{4}\sqrt{10}t\right) \\
y(t) &= \frac{1}{5}\sqrt{5}\sin\left(\frac{5}{4}\sqrt{2}t\right)\sin\left(\frac{1}{4}\sqrt{10}t\right) - \cos\left(\frac{5}{4}\sqrt{2}t\right)\cos\left(\frac{1}{4}\sqrt{10}t\right)
\end{aligned}
$$





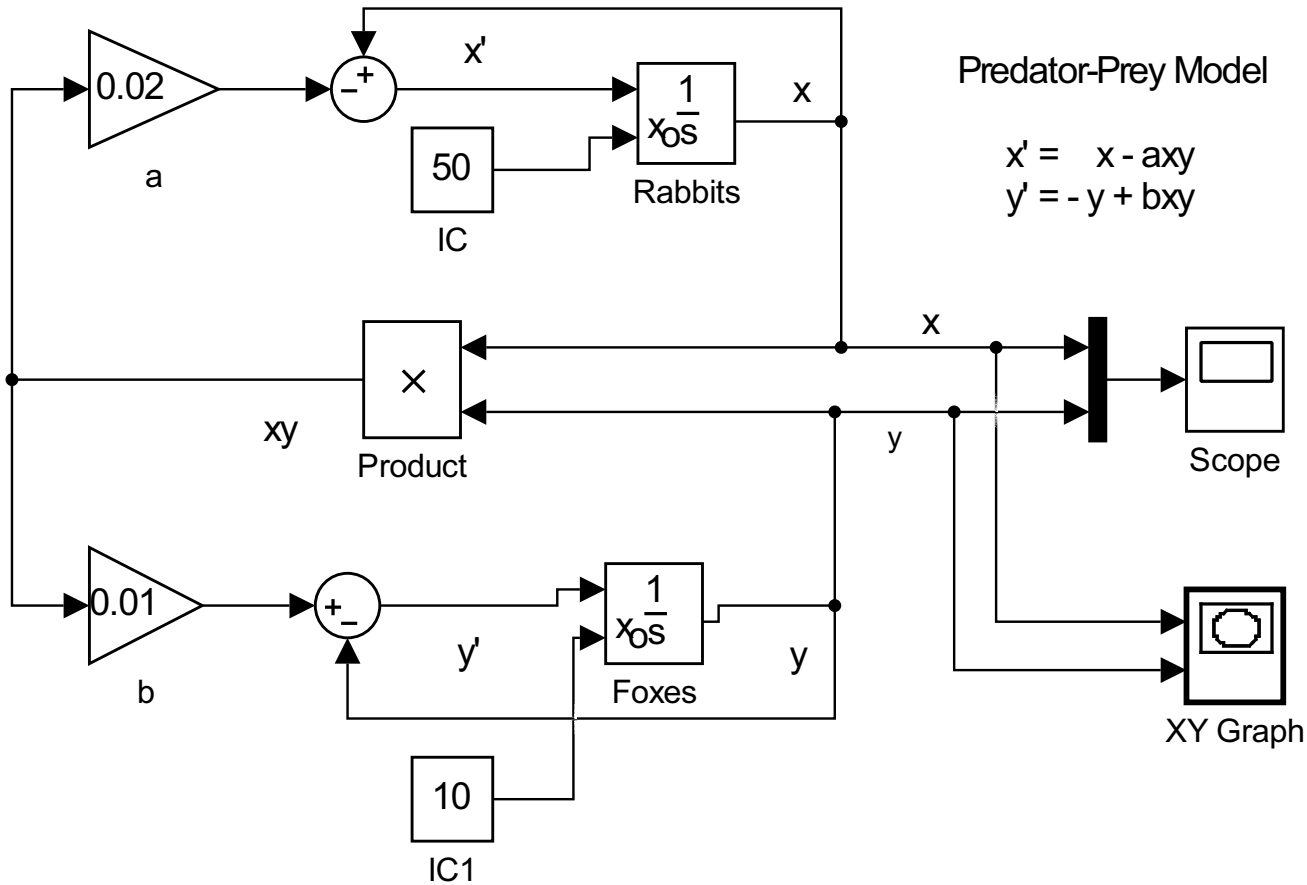
Linear Systems - Matrix Form

$$\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



Applications - Lotka Volterra Predator-Prey Model

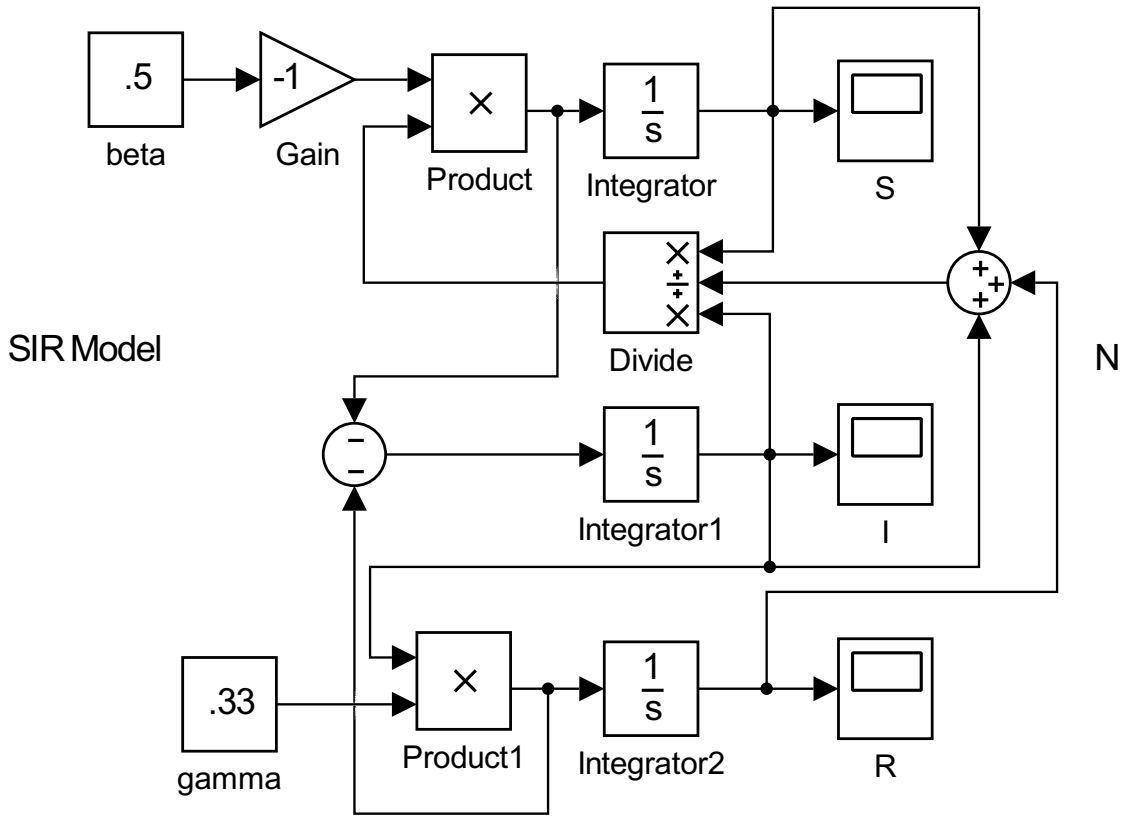
$$x' = x - axy \quad y' = -y + bxy.$$



Applications - SIR Epidemic Model

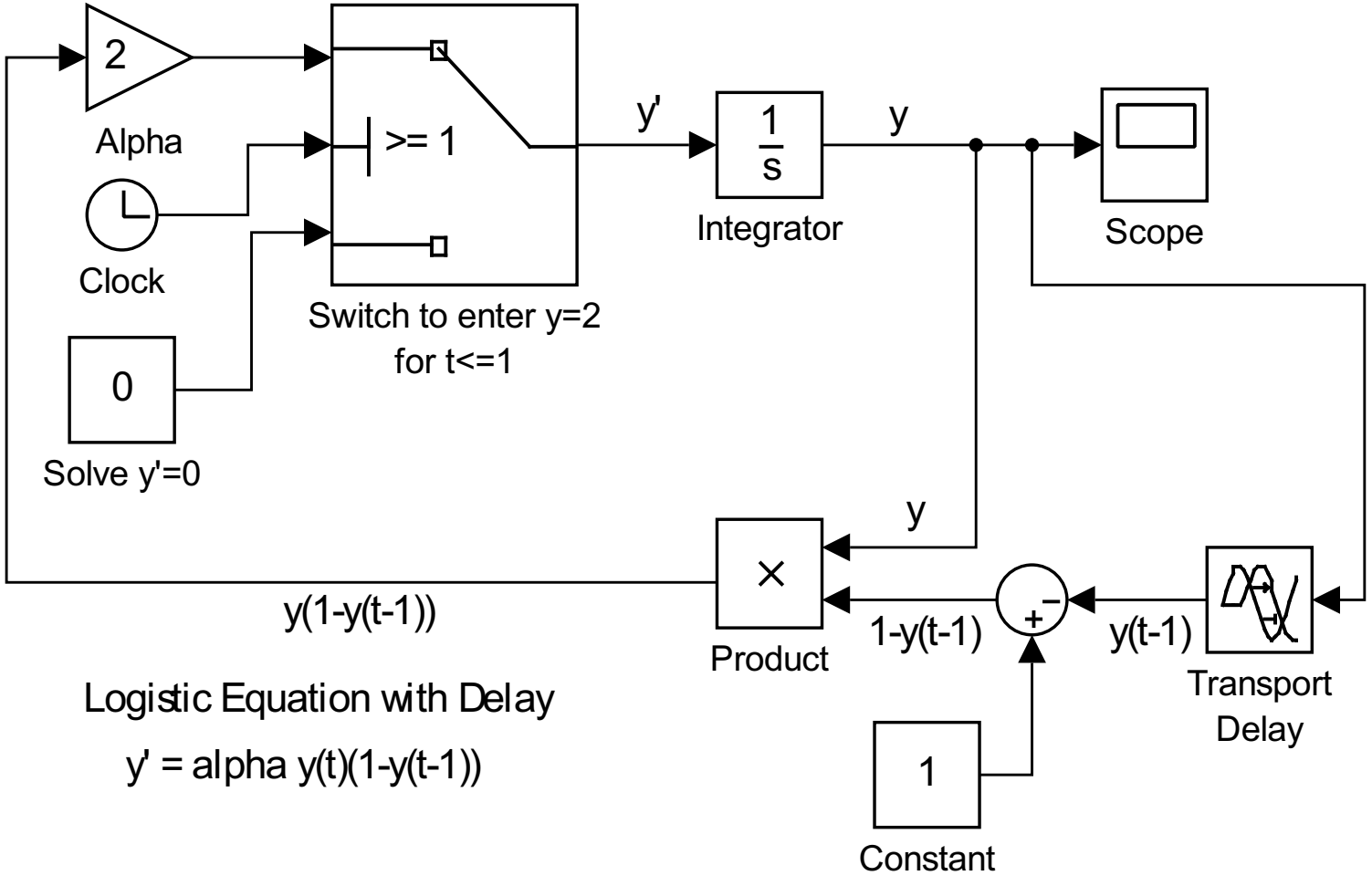
$$S' = -\beta \frac{SI}{N}, \quad I' = \beta \frac{SI}{N} - \gamma I, \quad R' = \gamma I.$$

where $N = S + I + R$.



Applications - Logistic Equation with Delay

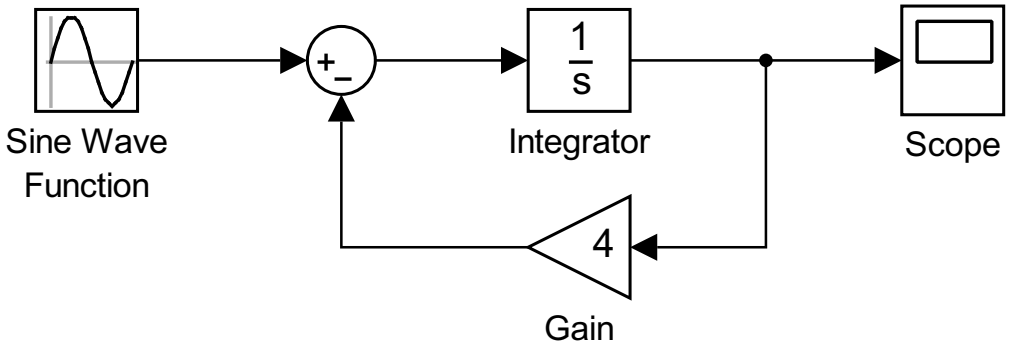
Solve $y' = ry(t)(1 - y(t - \tau))$ where $\tau > 0$ provides a delayed response.



Summary

Demo of Simulink Models for Simulation of Differential Equations

- Graphical Interface
- System Design
- Quick Entry to Applications
- Preparation to Engineering Applications



Thank You!

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