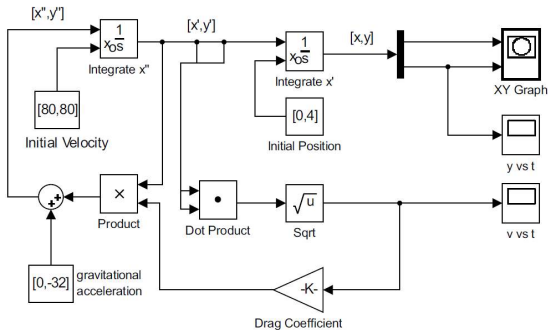


Solving Differential Equations with Simulink

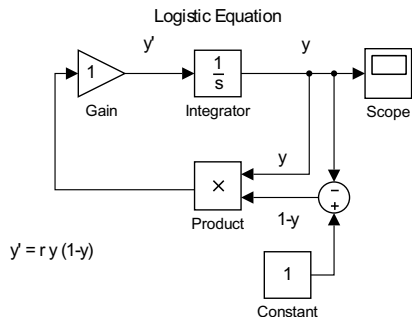
Dr. R. L. Herman
UNC Wilmington, Wilmington, NC



Outline

- 1 Simulink
- 2 Solution of ODEs
- 3 First Order Differential Equations
- 4 Second Order Differential Equations
- 5 Linear Systems
- 6 Nonlinear Systems
- 7 Summary

- What is Simulink
 - Graphical environment for designing simulations
 - Product of Mathworks
 - Select and connect blocks
- Use in Differential Equations
 - Project component of class
 - Modeling applications



Solving a Differential Equation

Consider initial value problem:

$$\frac{dx}{dt} = f(x), \quad x(0) = x_0.$$

Solution

$$x(t) = x_0 + \int_0^t f(x(t)) dt.$$

Think of the solution as

$$x(t) = \int x'(t) dt.$$

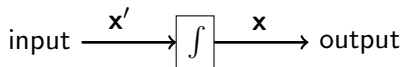


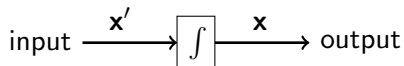
Figure : Schematic for a general system.

Simulink Model of a Differential Equation

Modeling

$$x(t) = \int x'(t) dt = \int f(x(t)) dt.$$

Schematic



Simulink Model

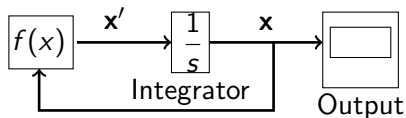
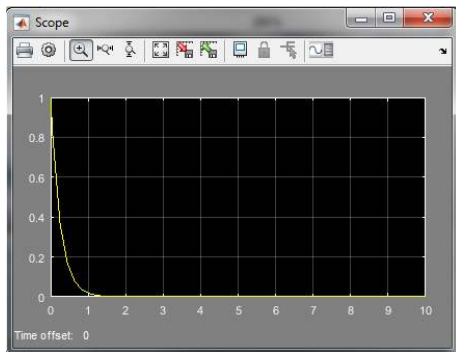
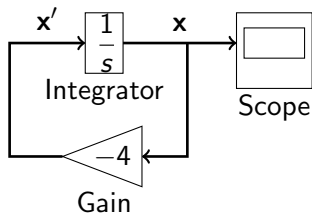


Figure : Model for solving $x' = f(x)$.

First Order Differential Equation - Example 1

Solve $x' = -4x$, $x(0) = 1$.



Simulink Workspace

The screenshot displays the Simulink workspace with several key components:

- Main Diagram:** A block diagram showing a feedback loop. A signal enters from the left, passes through a Gain block with a value of -4, then an Integrator block (1/s), and finally a Scope block. The signal then loops back to the input of the Gain block.
- Configuration Parameters:** A panel on the right showing simulation settings. The Solver is set to 'ode45 (Dormand-Prince)'. The Solver options include: Type: Variable-step, Max step size: auto, Min step size: auto, Initial step size: auto, and Number of consecutive min steps: 1. The Solver parameters are: Relative tolerance: 1e-3, Absolute tolerance: auto, and Shape preservation: Disable All. The Algorithm is set to Nonadaptive and the Signal threshold is auto (1000).
- Simulink Library Browser:** A central window showing a list of blocks. The 'Integrator' block (1/s) is highlighted in the 'Commonly Used Blocks' section.
- Scope:** A plot window showing the output of the Scope block. The x-axis is labeled 'Time offset: 0' and ranges from 0 to 10. The y-axis ranges from 0 to 1. The plot shows a curve that starts at 1.0 and decays exponentially towards 0.

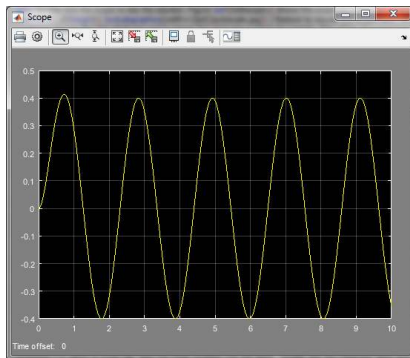
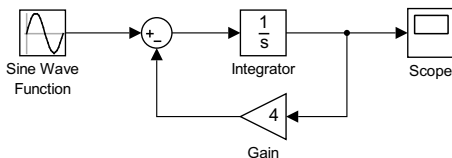
Scilab's Xcos Workspace

The screenshot displays the Scilab 5.5.2 Xcos workspace, which is a graphical environment for building and simulating control systems. The interface is divided into several windows:

- Palette browser - Xcos:** Located in the top-left, it contains a tree view of block categories (e.g., Commonly Used Blocks, Continuous time systems) and a grid of available blocks such as ANDBLK, BIGSOM_f, CMSCOPE, and CONST_m.
- Graphic window number 20004:** Located in the top-right, it shows a plot of a signal over time. The x-axis ranges from 0 to 10, and the y-axis ranges from -2 to 4. The signal is a smooth, periodic wave oscillating around zero.
- *Untitled - 8:14:42 AM - Xcos:** Located in the bottom-left, it shows a block diagram. A sine wave block is connected to a summing junction (Σ), which is followed by an integrator block ($1/s$). The output of the integrator is fed back to the summing junction and also passes through a gain block (-4) before being fed back to the summing junction. The final output is connected to a scope block (CMSCOPE).
- Scilab 5.5.2 Console:** Located in the bottom-right, it displays the startup execution log: "Startup execution: loading initial environment" followed by a prompt "-->". It also includes a file browser, a variable browser, and a command history window.

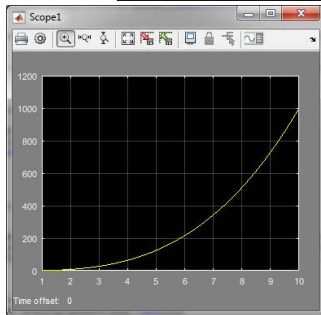
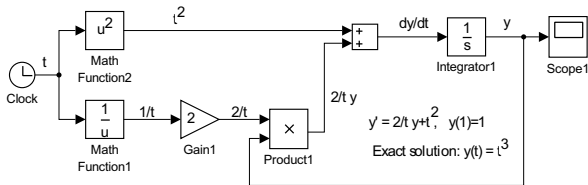
First Order Differential Equation - Example 1

Solve $x' = 2 \sin 3t - 4x$, $x(0) = 0$.



First Order Differential Equation - Example 3

Solve $y' = \frac{2}{t}y + t^2$, $y(1) = 1$.



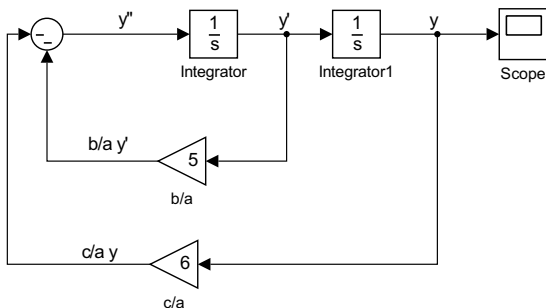
Second Order ODEs

Solve $ay'' + by' + cy = 0$, $y(0) = y_0, y'(0) = v_0$.

$$y = \int y' dx, \quad y' = \int y'' dx,$$

$$y'' = -\frac{b}{a}y' - \frac{c}{a}y.$$

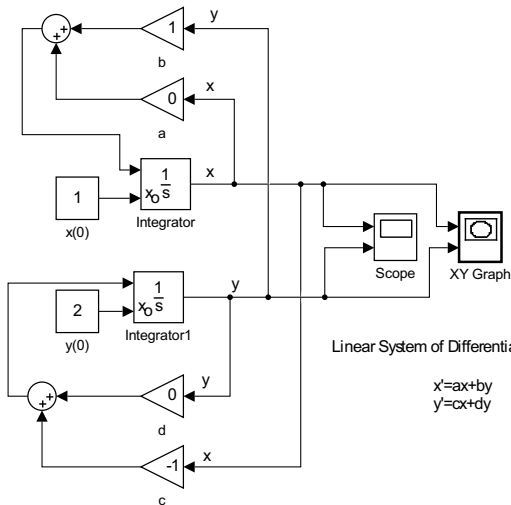
Second Order Constant Coefficient ODE



Linear Systems of Differential Equations

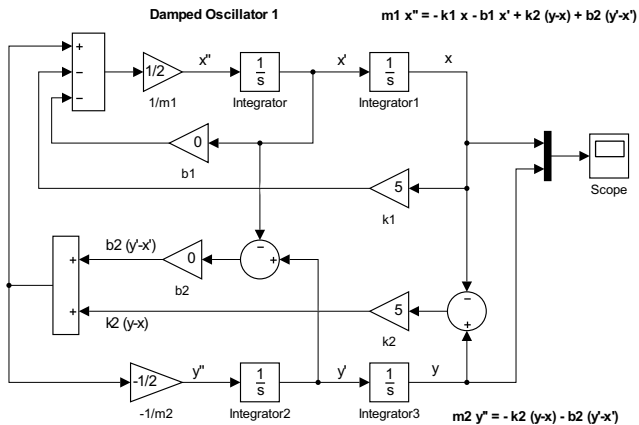
$$x' = ax + by$$

$$y' = cx + dy.$$



Coupled Oscillators

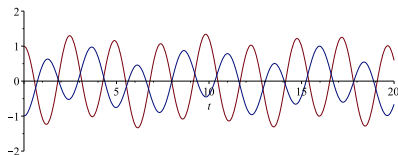
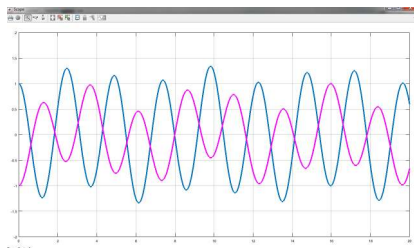
$$x'' = -k_1 x - b_1 x' + k_2(y - x) + b_2(y' - x')$$
$$y'' = -k_2(y - x) - b_2(y' - x').$$



Coupled Oscillators - Solutions

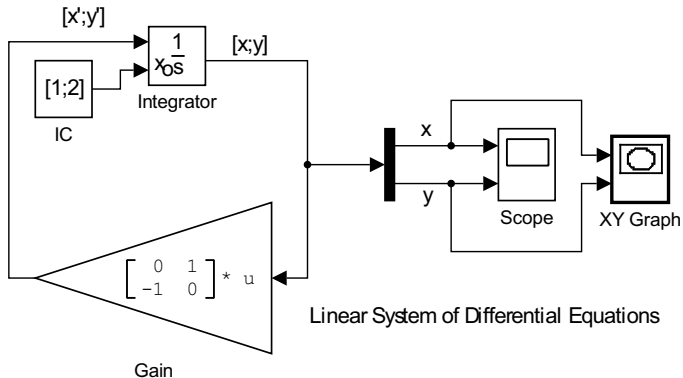
$$x(t) = \cos\left(\frac{5}{4}\sqrt{2}t\right) \cos\left(\frac{1}{4}\sqrt{10}t\right) - \frac{3}{5}\sqrt{5} \sin\left(\frac{5}{4}\sqrt{2}t\right) \sin\left(\frac{1}{4}\sqrt{10}t\right)$$

$$y(t) = \frac{1}{5}\sqrt{5} \sin\left(\frac{5}{4}\sqrt{2}t\right) \sin\left(\frac{1}{4}\sqrt{10}t\right) - \cos\left(\frac{5}{4}\sqrt{2}t\right) \cos\left(\frac{1}{4}\sqrt{10}t\right)$$



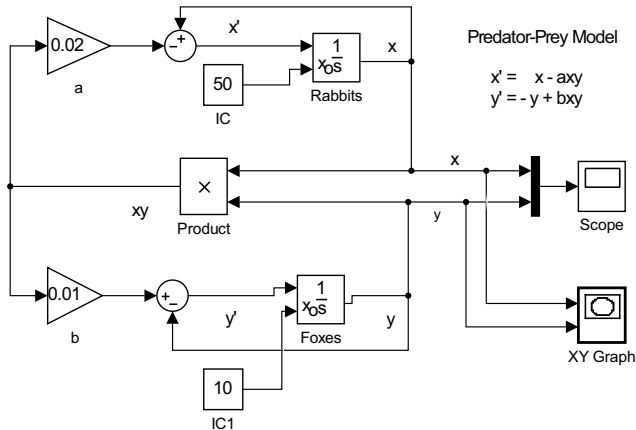
Linear Systems - Matrix Form

$$\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



Applications - Lotka Volterra Predator-Prey Model

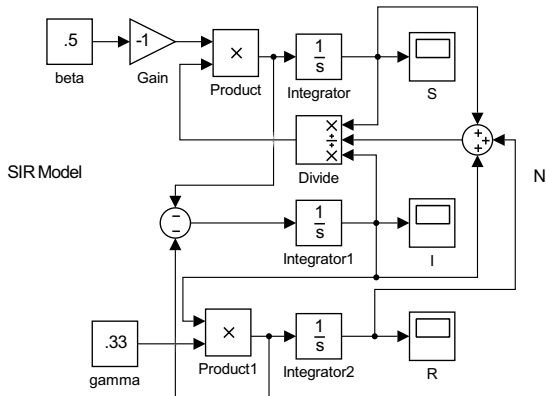
$$x' = x - axy \quad y' = -y + bxy.$$



Applications - SIR Epidemic Model

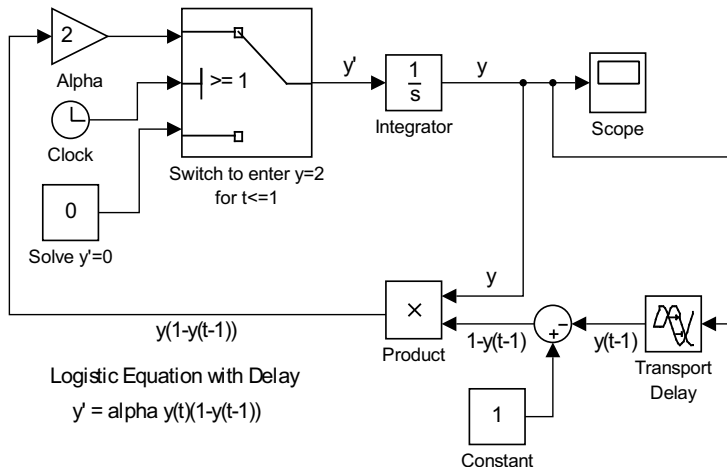
$$S' = -\beta \frac{SI}{N}, \quad I' = \beta \frac{SI}{N} - \gamma I, \quad R' = \gamma I.$$

where $N = S + I + R$.



Applications - Logistic Equation with Delay

Solve $y' = ry(t)(1 - y(t - \tau))$ where $\tau > 0$ provides a delayed response.

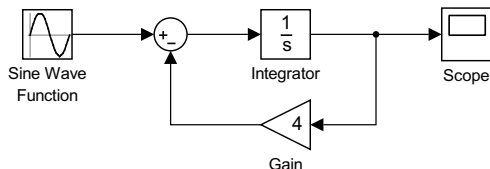


Logistic Equation with Delay
 $y' = \alpha y(t)(1 - y(t - \tau))$

Summary

Demo of Simulink Models for Simulation of Differential Equations

- Graphical Interface
- System Design
- Quick Entry to Applications
- Preparation to Engineering Applications



Thank You!
hermanr@uncw.edu