

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- All short answers and essays should be responded to with full sentences conveying thoughtful responses.
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space. **No other paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient rationale for full credit.

Use proper significant digits and units.

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	16	
2	16	
3	20	
4	13	
Total	65	

Some Constants - $M_{sun} = 1.989 \times 10^{30} \text{ kg}$, $M_{earth} = 5.98 \times 10^{24} \text{ kg}$, $1 \text{ Mpc} = 3.086 \times 10^{22} \text{ m}$,
 $G = 6.673 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$, $h = 6.626 \times 10^{-34} \text{ Js}$, $\epsilon_{c,0} = 8.3 \times 10^{-10} \text{ Jm}^{-3}$,

1. Provide the following based on our current observations/theory:
 - a. How old is the universe? _____
 - b. How many galaxies are there? _____
 - c. How "big" is the universe? _____
 - d. What is the current CMB temperature? _____

2. Hubble first obtained H_0 as $550 \text{ km s}^{-1} \text{ Mpc}^{-1}$. What would this give for the age of the universe in years?

3. Describe the following:
 - a. Isotropic

 - b. Vacuum energy

 - c. Steady State Model

4. Compute the Schwarzschild radius of the sun.

5. Why is the sky dark? (What is this paradox? What assumptions are needed to resolve the paradox?)

6. One of the highest measured red shifts is 6.96 for IOK-1 in April 2006. (Assume nonrelativistic speeds.)
 - a. If the star emitted a wavelength of 550 nm, then what wavelength was observed?

 - b. How far away might IOK-1 be?

 - c. How fast is it moving?

 - d. What was $a(t_e)$ for the universe at a time t_e when the light was emitted?

7. Explain Einstein's Equivalence Principle.

8. Use G , \hbar and c to construct a characteristic length, the Planck length.

9. Assume the density of the universe is $\rho \approx 10^{-28} \text{ kg m}^{-3}$ and the universe is 10^{10} years old. If there are 10^{11} stars per galaxy and the sun is a typical star, give an estimate of the number of galaxies in the universe.

10. Provide line elements for the following:

a. 2D spatial line element in polar coordinates.

b. Minkowski spacetime in spherical coordinates.

c. Schwarzschild metric

d. Robertson Walker line element for a homogeneous, isotropic spacetime.

11. Consider the Friedmann equation: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} \varepsilon - \frac{\kappa c^2}{R_0^2 a^2}$. Rewrite this equation at the current time in terms of H_0 and Ω_0 .

12. Consider the cosmic microwave background temperature. Using $\varepsilon_\gamma = aT^4$,

$$\alpha = 7.56 \times 10^{-16} \text{ J m}^{-3} \text{ K}^{-4}, \text{ and } n_\gamma = \beta T^3, \beta = 2.03 \times 10^7 \text{ m}^{-3} \text{ K}^{-3},$$

a. Determine the wavelength of the microwave radiation.

b. Find the contribution to Ω due to the background photons.

13. GPS satellites are located at 26560 km from the center of the Earth, moving at 3.9 km/s. These orbits have a period of motion of 11 hours and 58 minutes. Their operation depends on time measurements, but you know that there are relativistic differences between the time recorded on the satellite and that on the Earth.

- a. Due to special relativity, would the clocks tick faster or slower on the satellite? _____
- b. Due to general relativity, would the clocks tick faster or slower on the satellite? _____
- c. What is the ratio of time interval measurements ($\Delta\tau_s / \Delta\tau_E$)
 - i. Due to special relativity?

ii. Due to general relativity?

- d. Over the course of a day, what is the total time difference between a clock on the Earth and one on a GPS satellite.

14. The equation of state takes the form $P = w\varepsilon$. What is w for

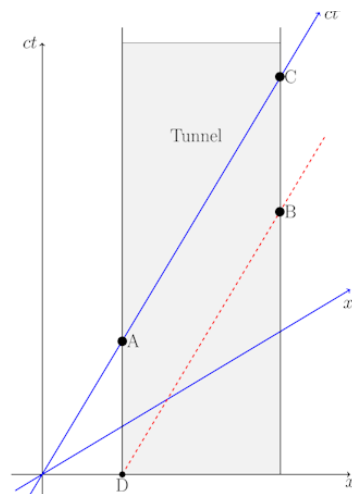
- a. A photon gas? _____
- b. Dark energy? _____

Added questions:

15. A triangle is drawn on a ball of radius 4.00 m. The sum of the interior angles is 270° . What is the area of the triangle?
16. A particle is measured to have a kinetic energy that is four times its rest mass energy. How fast is the particle moving?
17. Let $(x, ct) = (200, 350)$ m in System S. In a system moving at $0.6c$ with respect to S, what are the measured coordinates (x', ct') ?
18. The Sears Tower in Chicago was the tallest building from 1973 to 1998. It is 442m to the roof and 527m to the top of the antenna. A 931 MHz signal is sent to the ground. What is the change in the frequency of the signal when received on the ground?
19. (8 pts) A train of rest length 240 meters travels at $0.6c$ through a tunnel which has rest length 360 meters

- D: The front of the train enters tunnel.
- B: The front of the train leaves tunnel.
- A: The back of the train enters the tunnel.
- C: The back of the train leaves the tunnel.

Answer the following for the given frame: Use units with $c=1$.



- i. The coordinates (x, t) of B in the tunnel frame of reference.
 - ii. The coordinates of A in the train frame of reference.
 - iii. Does event A occur before or after B in the tunnel frame of reference? _____
 - iv. Does event A occur before or after B in the train frame of reference? _____
20. Need to review Chap 1-2 of Owocki for additional questions.