Cosmology Review II

1. People
    a. Vesto Slipher
    b. Henrietta Leavitt
    c. E. Hubble
    d. Sir Arthur Eddington
    e. Henri Poincaré
    f. Chandrasekhar
    g. William Thomson aka Lord Kelvin
2. Objects
    a. Cepheids
    b. M31
    c. Supernovae
    d. CMB
    e. White-brown-red dwarfs, neutron stars, black holes, MACHOs, WIMPs
    f. Barnard's star
    g. Hipparcos
3. Terms
    a. Hubble time, Hubble length
    b. Horizon distance
    c. Proper distance
    d. Luminosity distance
    e. Planck scales, $\ell_P, M_P, t_P, E_P, T_P$.
    f. Chandrasekhar limit
    g. Baryonic
    h. Deceleration parameter
    i. Standard candle, standard yardstick
    j. Lookback time
    k. Bolometer
    l. Luminosity, flux
    m. Surface brightness
    n. Big Crunch, Big Freeze, Big Bounce
    o. Loitering universe
    p. Benchmark Model
    q. Density is destiny
    r. Big Chill, Big Crunch, Big bounce, Loitering
    s. Hyperbolic function substitution
    t. Beta ($\beta^{\pm}$) decay
    u. Polytropic
    v. Hydrostatic equilibrium
4. Hubble Law $v = H_0 r$, $z \approx \frac{v}{c}$, Hubble parameter $H(t) = \frac{\dot{a}(t)}{a(t)}$
5. Hubble time H_0^{-1}, Hubble length $\frac{c}{H_0}$

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6. Steady State Model – H_0, ρ_0 constant => $\frac{dr}{dt} = H_0 r$

7. Horizon distance $d_p(t) = c\int_0^{t_0} \frac{dt}{a(t)}$

8. Red shift $1+z = \frac{1}{a(t_e)}$, $a(t_0) = 1$.

9. Friedmann Equation

   a. $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2 a^2}$

   b. $H_0^2 = \frac{8\pi G}{3c^2}\varepsilon_0 - \frac{\kappa c^2}{R_0^2}$,

10. Energy density

    a. $\varepsilon_w = \varepsilon_{w,0} a^{-3(1+w)} \Rightarrow \varepsilon_m, \varepsilon_r$

    b. $\dot{a}^2 = \frac{8\pi G}{3c^2}\sum_w \varepsilon_{w,0} a^{-1-3w} - \frac{\kappa c^2}{R_0^2}$

11. Density parameters

    a. $\Omega_{CMB} = 5.35\times 10^{-5}$, $\Omega_\nu = 0.681\Omega_{CMB}$, $\Omega_r = 9.0\times 10^{-5}$

    b. $\Omega_m = 0.31, \Omega_\Lambda = 0.69$ (Benchmark)

12. Models

    a. Curvature Only (Zero, positive negative curvature – what happens? $a = ?, t_0 = ?, t_e = ?$)

    b. $a_{rm}, a_{m\Lambda}$

    c. Solve for $d_p(t_0) = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$, $d_p(t_e) = \frac{d_p(t_0)}{1+z}$

    d. Spatially Flat Universes

       i. $a(t) = \left(\frac{t}{t_e}\right)^{2/(3+3w)}$, $t_0 = \frac{2}{3(1+w)}H_0^{-1}$, $\varepsilon(t) = \frac{3c^2}{8\pi G}H_0^2\left(\frac{t}{t_0}\right)^{-2}$

       ii. Lambda Only

          1. $\dot{a}^2 = \frac{8\pi G\varepsilon_\Lambda}{3c^2}a^2 \Rightarrow \dot{a} = H_0 a \Rightarrow a(t) = e^{H_0(t-t_0)}$

    e. Multiple-Component Universes

       i. $\frac{\kappa}{R_0^2} = \frac{H_0^2}{c^2}(1-\Omega_0)$,

       ii. $\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$

       iii. $H_0 t = \int_0^a \frac{da}{\sqrt{\frac{\Omega_{r,0}}{a^2} + \frac{\Omega_{m,0}}{a} + \Omega_{\Lambda,0}a^2 + 1 - \Omega_0}}$

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iv. Determining eras of dominant components (Epochs), $a_{rm}, a_{m\Lambda}$, t_{crunch}, etc.

v. Fates of the universe in matter plus curvature plus Λ universes.

vi. Scale factor vs time for different universes.

13. Cosmological Parameters

a. $a(t) \approx 1 + H_0(t-t_0) - \frac{1}{2} q_0 H_0^2 (t-t_0)^2$, $H(t) = \frac{\dot{a}(t)}{a(t)}$, $q_0 = -\left.\frac{\ddot{a}a}{\dot{a}^2}\right|_{t_0} = -\left.\frac{\ddot{a}}{aH^2}\right|_{t_0}$

b. $q_0 = \Omega_{r,0} + \frac{1}{2}\Omega_{m,0} - \Omega_{\Lambda,0}$

c. Look back time $t_0 - t_e \approx H_0^{-1}\left[z - \left(1 + \frac{q_0}{2}\right) z^2\right]$

d. $d_p(t_0) \approx \frac{c}{H_0} z \left[1 - \frac{1+q_0}{2} z\right]$

14. Luminosity

a. flux $f = \frac{L}{4\pi S_\kappa^2(r)(1+z)^2}$, $F = \frac{L}{4\pi d^2}$, $d_L = \left(\frac{L}{4\pi f}\right)^{1/2}$

b. $F_\Theta = 1.4\ \mathrm{kW/m^2}, L_\Theta = 4 \times 10^{26}\ \mathrm{W}$

c. Surface Brightness, $I = \frac{F}{\Omega}$

d. Luminosity distance $d_L = S_\kappa^2(r)(1+z), d_L = d(t_0)(1+z), [\kappa = 0]$

e. Parallax - $d_{pc} = \frac{1}{\theta_{arc\sec}}$

f. Angular diameter distance $d_A = \frac{\ell}{\delta\theta}, d_L = \frac{d_L}{(1+z)^2} = \frac{d_p(t_0)}{(1+z)}, [\kappa = 0]$

g. RW Universe – wavelength growth, energy drop, time dilates - $f = \frac{L}{4\pi S_k^2(r)(1+z)^2}$

h. Standard Candles – Cepheids, Supernovae Type Ia - $L = 4 \times 10^9 L_{sun}$

i. $d_L \propto \overline{f}$

j. Apparent magnitude $m = -2.5 \log_{10} \frac{f}{f_x}$, Distance modulus $m - M = 5 \log_{10} \frac{d}{10pc}$

k. In terms of Luminosity: $m = 4.8 - 2.5 \log_{10} \frac{L}{L_\Theta} + 5 \log_{10} \frac{d}{10pc}$

l. Absolute magnitude $M = -2.5 \log_{10} \frac{L}{L_x} = m - 5 \log_{10} \frac{d_L}{1Mpc} - 25$

15. Surface Temperature

a. EM waves, photon energy and momentum

b. Blackbody radiation

c. Wein's Displacement Law, $\lambda_{\max} \simeq \frac{500\mathrm{nm}}{T / T_\Theta}$

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d. $T_{\Theta} = 5800K$

e. $B(T) = \sigma T^4 / \pi, F_* = \dfrac{L}{4\pi R^2} = \sigma T^4$, Stefan-Boltzmann Law

16. Composition of Stars, relative abundances
17. Stellar classification (OBAFGKM) and classes (I-V),Hertzsprung-Russell diagram
18. Main sequence, Reg giants, giants, white dwarfs, …
19. Doppler Effect, $\dfrac{\Delta\lambda}{\lambda} = \dfrac{v_r}{c}$
20. Escape velocity $v = \sqrt{\dfrac{2GM}{R}}$, orbital velocity $v = \sqrt{\dfrac{GM}{r}}$
21. Stellar Age estimations, processes (Chemical, gravitational, Proton-proton)
22. $4\,{}_1H^{+} \rightarrow {}_4He^{+2} + \nu + 2e^{+} + E_{\gamma}$
23. Stellar space velocities, proper motion, radial and tangential components
24. Binary systems, orbital periods, mass determination.
25. Kepler's 3rd Law - $T^2 = a^3$ in years and au's.
26. Spectroscopic, eclipsing, astrometric binaries.
27. Stellar Equilibrium, polytropic models