Cosmology Review I

1. People
    a. Olbers Paradox (1826)
    b. Einstein
    c. Planck
    d. Slipher
    e. Hubble
    f. Bondi, Gold, Hoyle – Steady State Model
    g. Penzias and Wilson (1965) - CMB
    h. Friedmann, Lemaitre, Robertson, Walker
2. Units – AU, pc, Mpc, Planck units, light year, flux, luminosity, Size of Milky Way
3. Astronomical Distances, parallax $\frac{\alpha}{\text{arcsec}} = \frac{s/\text{au}}{d/\text{pc}}$, angular size $\sin\frac{\alpha}{2} = \frac{R}{d}$
4. Solid angle $\Omega = \frac{A}{d^2} \approx \pi\alpha^2$
5. Olbers Paradox (1826)
6. Isotropic, Homogeneous, Geodesics, Equivalence Principle
7. Hubble Law $v = H_0 r,\ z \approx \frac{v}{c}$, Hubble parameter $H(t) = \frac{\dot{a}(t)}{a(t)}$
8. Hubble constant $H_0 = 68 \pm 2\ \text{km s}^{-1}\ \text{Mpc}^{-1}$,
9. Hubble time $H_0^{-1} = 14.38 \pm 0.42\ \text{Gyr},$, Hubble length $\frac{c}{H_0} = 4380 \pm 130\ \text{Mpc}$
10. Steady State Model – H_0, ρ_0 constant => $\frac{dr}{dt} = H_0 r$
11. Blackbody radiation - $\varepsilon_\gamma = \alpha T^4, n_\gamma = \beta T^3$
12. CMB $2.7255 \pm 0.006\ \text{K}$
13. Curvature $\alpha + \beta + \gamma = \pi + \frac{\kappa A}{R_0^2}$
14. Line elements
    a. Minkowski metric $ds^2 = -c^2 dt^2 + dr^2 + S_k(r)^2 d\Omega^2,\ \ d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$
        i. Lorentz transformation
        ii. $dx' = \gamma(dx - vdt), cdt' = \gamma(cdt - \beta dx)$, $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$, $\beta = \frac{v}{c}$
        iii. Time dilation, Length Contraction
        iv. Light cones, past cone, future cone, null cone

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b. $ds^2 = dr^2 + S_k(r)^2 d\Omega^2$, $S_k(r) = \begin{cases} R\sin\frac{r}{R}, & \kappa = +1 \\ r, & \kappa = 0 \\ R\sinh\frac{r}{R}, & \kappa = -1 \end{cases}$

c. $ds^2 = -c^2 d\tau^2 = -\left(1-\frac{r_c}{r}\right)c^2dt^2 + \left(1-\frac{r_c}{r}\right)^{-1}dr^2 + r^2 d\Omega^2$, $r_c = \frac{2GM}{rc^2}$

d. FRLW: $ds^2 = -c^2dt^2 + a(t)^2[dr^2 + S_k(r)^2 d\Omega^2]$

e. Gravitational redshift $\Delta\tau_B = \left(1 - \frac{\Phi_A - \Phi_B}{c^2}\right)\Delta\tau_A$, $\Phi = gh$, $\Phi = -\frac{GM}{R}$

Signals received at lower potential are received at faster rate.

15. Proper distance $d_p(t) = a(t)r$ Recession speed $v_p(t_0) = \dot{d}_p(t_0) = H_0 d_p(t_0)$

16. Red shift $1+z = \frac{1}{a(t_e)}$, $a(t_0) = 1$. $z = \frac{\lambda_o - \lambda_e}{\lambda_e}$

17. Newtonian dynamics $\nabla^2\Phi = 4\pi G\rho$

18. Friedmann Equation $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2 a^2}$

a. $H_0^2 = \frac{8\pi G}{3c^2}\varepsilon_0 - \frac{\kappa c^2}{R_0^2}$, $\frac{\kappa}{R_0^2} = \frac{H_0^2}{c^2}(1-\Omega_0)$,

b. Density parameter $\Omega(t) = \frac{\varepsilon(t)}{\varepsilon_0(t)}$, Critical density $\varepsilon_c(t) = \frac{3c^2}{8\pi G}H^2(t)$

19. Equations of State

a. Ideal gas $P = \frac{\rho}{\mu}kT \approx \frac{kT}{\mu c^2}\varepsilon$

b. $P = \omega\varepsilon$, $\omega = \begin{cases} 0, & \text{nonrelativisitc matter} \\ 1/3, & \text{radiation} \\ -1, & \text{exotic, } \Lambda \end{cases}$

20. Fluid Equation $\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$, Acceleration Equation $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P)$

21. Cosmological Constant $\varepsilon_\Lambda = \frac{c^2}{8\pi G}\Lambda = -P_\Lambda$

a. $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2 a^2} + \frac{\Lambda}{3}$,

b. $\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$, $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P) + \frac{\Lambda}{3}$