PHY 490 Astrophysics and Cosmology | Solution
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Train Problem

A relativistic train of rest length 240 meters travels at $0.6c$ through a tunnel which has rest length 360 meters. In Figure 1 the world lines for the tunnel openings are drawn and the world line of the front of the train is the dotted red line. Let S_{tunnel} be the tunnel frame with coordinates (x, t) and let Strain be the train coordinates (x', t'). We set the origin as the event the back of the train location just as the front end enters the tunnel opening.

Given the above information, we can locate the points

F1: The front of the train enters door 1.
F2: The front of the train passes door 2.
B1: The back of the train enters the tunnel.
B2: The back of the train leaves the tunnel.

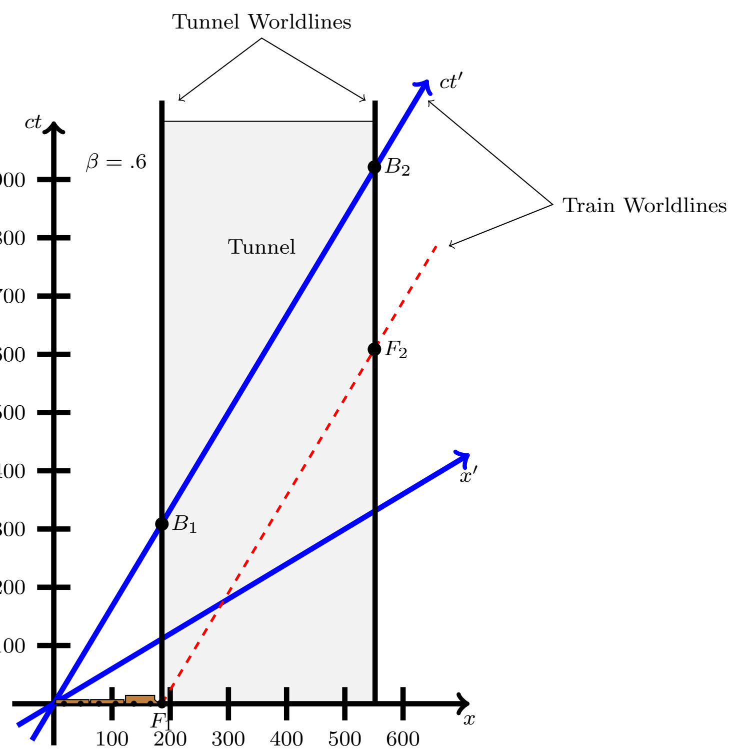

Figure 1: The Minkowski diagram for the problem of a train going through a tunnel.

First, we note that $\beta = 0.6$ and $\gamma = 1.25$. Now, the length of the train is 240 m. This is in the rest frame of the train. So, $L_0 = 240$m and is measured along the blue x'-axis. Then, length contraction gives $L = L_0/\gamma = 192$m, which is measure along the x-axis. This means that the tunnel entrance goes from F_1, $x = 192$m, to $x = 192 + 360 = 552$m, which is also the x-coordinate of F_2 and B_2.

Also, the back of the train now travels 192m from the origin to the red line through B_1 at speed $0.6c$. The time it takes is $ct = 192\text{m}/.6 = 320\text{m}$. These values are placed in Table 1.

	x	ct	x'	ct'
F_1	192	0	–	–
F_2	552	–	–	–
B_1	192	320	–	–
B_2	552	–	–	–

Table 1: First pass at values in meters of the coordinates for the given points.

Now we proceed to add more values to the table. We can reason using the Lorentz transformations, time dilation, or length contraction. Turning to the front of the train, F_2, as it leaves the tunnel it has been traveling the length of the tunnel at a known speed. Therefore, $ct = 360\text{m}/.6 = 600\text{m}$. Similarly, the back of the train travels from the origin to the red line passing through B_2. So, $ct = 552\text{m}/.6 = 920\text{m}$. We can update the table and obtain Table 2.

	x	ct	x'	ct'
F_1	192	0	–	–
F_2	552	600	–	–
B_1	192	320	–	–
B_2	552	920	–	–

Table 2: Second pass at values in meters of the coordinates for the given points.

We have all of the unprimed values, so now we can find the primed variable values using the Lorentz transformation,

$$\begin{aligned} x' &= \gamma(x - \beta ct) \\ ct' &= \gamma(ct - \beta x). \end{aligned} \tag{1}$$

For example, for F_2,

$$\begin{aligned} x' &= 1.25(552 - .6(600)) = 192\gamma = 240 \\ ct' &= 1.25(600 - .6(552)) = 336. \end{aligned} \tag{2}$$

The rest of the table can be filled this was as shown in Table 3.

	x	ct	x'	ct'
F_1	192	0	240	-144
F_2	552	600	240	336
B_1	192	320	0	256
B_2	552	920	0	736

Table 3: Second pass at values in meters of the coordinates for the given points.