

Name: \_\_\_\_\_

Justify each answer by providing supporting details. Answers with incomplete or incorrect justification may not receive full credit.

1. Show that integers  $n \geq 1$ ,

$$2 + 5 + 8 + \cdots + (3n - 1) = \frac{n(3n + 1)}{2}.$$

2. Show that for all integers  $n \geq 1$ ,

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n + 1)(2n + 1)}{6}.$$

3. Consider the sequence  $(a_n)$  defined recursively by

$$a_1 = 3 \text{ with } a_{n+1} = 2a_n + 1.$$

Prove by induction that

$$a_n = 2^{n+1} - 1 \text{ for all } n \geq 1.$$

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