Homework #2, Problem 5, 7, 9 in chapter 3

## 3.5) OVERHAUSER EFFECT

Let $U_0$ be the energy of the reservoir when the energy of the system is zero. Then, when the system has the energy $\varepsilon$, the reservoir has, by our supposition, the energy $U_0 - \varepsilon + \alpha\varepsilon = U_0 - (1-\alpha)\varepsilon$. The probability $P(\varepsilon_s)$ to find the system in a particular state with energy $\varepsilon_s$ is then proportional to the number of states of the reservoir with the energy $U_0 - (1-\alpha)\varepsilon_s$,

$$P(\varepsilon_s) \propto g_R[U_0-(1-\alpha)\varepsilon_s],$$

with the same proportionality factor for all states. Hence, instead of (2) and (3):

$$\frac{P(\varepsilon_1)}{P(\varepsilon_2)} = \frac{g_R[U_0-(1-\alpha)\varepsilon_1]}{g_R[U_0-(1-\alpha)\varepsilon_2]} = \frac{\exp\{\sigma_R[U_0-(1-\alpha)\varepsilon_1]\}}{\exp\{\sigma_R[U_0-(1-\alpha)\varepsilon_2]\}} \quad . \qquad (S1)$$

If the entropy of the reservoir is expanded about $U = U_0$, as in (7):

$$\sigma_R[U_0-(1-\alpha)\varepsilon] = \sigma_R(U_0) - (\partial\sigma_R/\partial U)(1-\alpha)\varepsilon + \ldots$$

If this is inserted into (S1), one obtains, instead of (9),

$$\frac{P(\varepsilon_1)}{P(\varepsilon_2)} = \frac{\exp[-(1-\alpha)\varepsilon_1/\tau]}{\exp[-(1-\alpha)\varepsilon_2/\tau]} \quad ,$$

which is equivalent to (91).

3.7) ZIPPER PROBLEM

(a) A state in which s links are open can be realized in only one way. Thus the partition function is

$$Z = 1 + \exp(-\varepsilon/\tau) + \exp(-2\varepsilon/\tau) + \ldots + \exp(-N\varepsilon/\tau) \quad .$$

$$= \sum_{s=0}^{N} x^s = \frac{1-x^{N+1}}{1-x} \text{ , where } x = \exp(-\varepsilon/\tau) \quad . \qquad (93)$$

(b) The average number of open links is

$$\langle s \rangle = \frac{1}{Z} \sum_{s=0}^{N} s x^s = x \frac{d}{dx} \log Z \quad . \qquad (S1)$$

If $\varepsilon >> \tau$, then $x << 1$, and we may neglect the term $x^{N+1}$ in (93) to obtain

$$\langle s \rangle = - x \frac{d}{dx} \log(1-x) = \frac{x}{1-x} = 1/[\exp(\varepsilon/\tau) - 1] \quad .$$

This is of the form of the Planck distribution.

3.9) PARTITION FUNCTION FOR TWO SYSTEMS

Every state s1 of system 1, with energy $\varepsilon_{s1}$, can be combined with every state s2 of system 2, with energy $\varepsilon_{s2}$, to form the different states s of the combined system, with energy $\varepsilon_s = \varepsilon_{s1} + \varepsilon_{s2}$:

$$Z(1+2) = \sum_{s} \exp[-\varepsilon_s/\tau] = \sum_{s1} \sum_{s2} \exp[-(\varepsilon_{s1}+\varepsilon_{s2})/\tau]$$

$$= \left[ \sum_{s1} \exp(-\varepsilon_{s1}/\tau) \right] \left[ \sum_{s2} \exp(-\varepsilon_{s2}/\tau) \right] \quad .$$

$$= Z(1)Z(2) \quad .$$