Homework-11, Problem 1, 2, 3, 5, 7 in chapter 8

8.1) <u>HEAT PUMP</u>

(a) If operating reversibly, this is just a Carnot engine with all flows reversed, hence (7) remains valid. As in a refrigerator, any entropy generated in a heat pump must leave at the higher temperature τ_h. Hence, as in (14),

$$Q_h/\tau_h = \sigma_h \geq \sigma_\ell = Q_\ell/\tau_\ell \quad ,$$

where the equals sign holds in the limit of reversible operation. Solved for Q_ℓ:

$$Q_\ell = \tau_\ell \sigma_\ell \leq \tau_\ell \sigma_h = (\tau_\ell/\tau_h) Q_h \quad .$$

Hence the energy required to drive the heat pump is

$$W = Q_h - Q_\ell \geq (1-\tau_\ell/\tau_h) Q_h = \eta_C Q_h \quad . \qquad \text{(S1)}$$

The reversible limit of this is equivalent to the claimed result; for irreversible operation more work is required.

(b) From (6), with τ_h and Q_h replaced by τ_{hh} and Q_{hh}:

$$W = (1-\tau_\ell/\tau_{hh}) Q_{hh} \quad .$$

Inserted into the reversible limit of (S1):

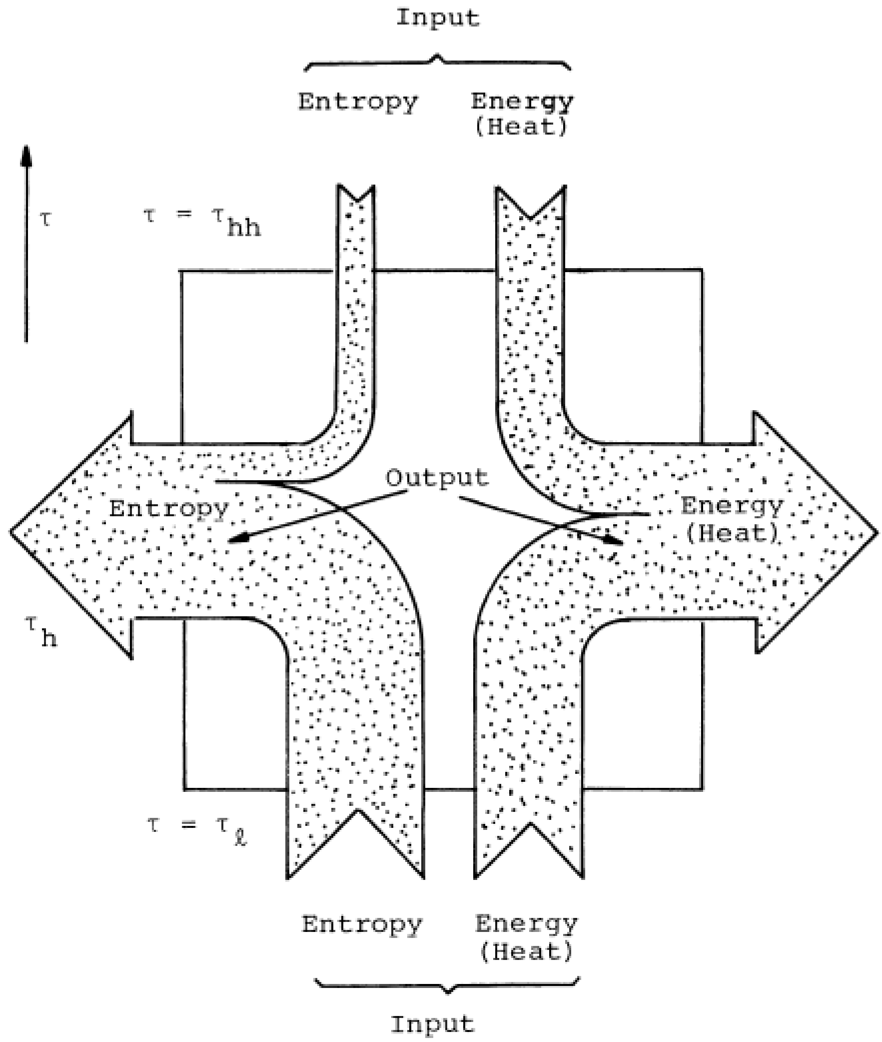
$$(1-\tau_\ell/\tau_{hh}) Q_{hh} = (1-\tau_\ell/\tau_h) Q_h \quad ,$$

$$Q_{hh}/Q_h = (1-\tau_\ell/\tau_h)/(1-\tau_\ell/\tau_{hh})$$

$$= (\tau_{hh}/\tau_h)(\tau_h-\tau_\ell)/(\tau_{hh}-\tau_\ell) = \eta_C(\tau_h,\tau_\ell)/\eta_C(\tau_{hh},\tau_\ell) \quad .$$

For our numerical example, $Q_{hh}/Q_h \cong 0.18$.

(c) See next page.



8.2) <u>ABSORPTION REFRIGERATOR</u>

(a) See figure above, which applies to this problem as well.

(b) From energy and entropy conservation:

$$Q_h = Q_{hh} + Q_\ell \ , \quad \sigma_h = Q_h/\tau_h = \sigma_{hh} + \sigma_\ell = Q_{hh}/\tau_{hh} + Q_\ell/\tau_\ell .$$

Multiply the entropy relation by τ_h and subtract from the energy relation:

$$0 = (1-\tau_h/\tau_{hh})Q_{hh} + (1-\tau_h/\tau_\ell)Q_\ell \quad ;$$

$$Q_\ell/Q_{hh} = (1-\tau_h/\tau_{hh})/(\tau_h/\tau_\ell - 1)$$

$$= (\tau_\ell/\tau_{hh})(\tau_{hh}-\tau_h)/(\tau_h-\tau_\ell) = \eta_C(\tau_{hh},\tau_h)\gamma_C(\tau_h,\tau_\ell) \quad .$$

A detailed description of the actual operating cycle of gas-heated absorption refrigerators is given in the Supplementary Material to this Chapter.

8.3) PHOTON CARNOT ENGINE

This problem draws upon (4.2) and (4.23), which we write in the abbreviated form

$$U = sV\tau^4 \quad , \quad \sigma = 4sV\tau^3/3 \quad , \qquad (4.20,23)$$

where $s = \pi^2/15\hbar^3c^3$. We designate by W_{ij} and Q_{ij} the work taken up by the system in going from "i" to "j".

(a) According to (4.23), $V\tau^3$ = constant in an isentropic process. Point 3 on the Carnot loop is isentropic with point 2 (see Figure 4.5), and point 4 is isentropic with point 1; hence

$$V_3\tau_\ell^3 = V_2\tau_h^3 \text{ or } V_3 = V_2\times(\tau_h/\tau_\ell)^3 \quad , \qquad (S1)$$

$$V_4\tau_\ell^3 = V_1\tau_h^3 \text{ or } V_4 = V_1\times(\tau_h/\tau_\ell)^3 \quad . \qquad (S2)$$

(b) From $đQ = \tau d\sigma$, with (4.23):

$$Q_h = Q_{12} = \tau_h \int_1^2 d\sigma = \tau_h\times(\sigma_2-\sigma_1) = (4s\tau_h^4/3)(V_2-V_1) \quad . \qquad (S3)$$

Also, from $đW = dU - đQ$, with (4.20):

$$W_{12} = U_2 - U_1 - Q_{12} = s\tau_h^4\times(V_2-V_1) - Q_{12}$$

$$= -(s\tau_h^4/3)(V_2-V_1) = -Q_h/4 \quad .$$

Hence, heat taken up and work done are not equal.

For the isothermal step 3 → 4 at $\tau = \tau_\ell$ we find similarly

$$W_{34} = -(s\tau_\ell^4/3)(V_4-V_3) = (s\tau_\ell\tau_h^3/3)(V_2-V_1) \quad ,$$

where we have utilized (S1,2) to replace V_3 and V_4 by V_2 and V_1. The combined work done <u>on</u> the system during the two isothermal stages is

$$W_{12} + W_{34} = -(s\tau_h^3/3)(\tau_h-\tau_\ell)(V_2-V_1) \quad . \tag{S4}$$

(c) No! We calculate the isentropic work as an energy difference, using (4.20), (S1) and (S2):

$$W_{23} = U_3-U_2 = s\times(V_3\tau_\ell^4-V_2\tau_h^4) = -sV_2\tau_h^3(\tau_h-\tau_\ell) \quad ,$$

$$W_{41} = U_1-U_4 = s\times(V_1\tau_h^4-V_4\tau_\ell^4) = sV_1\tau_h^3(\tau_h-\tau_\ell)$$

$$W_{23} + W_{41} = -s\tau_h^3(\tau_h-\tau_\ell)(V_2-V_1) \neq 0 \quad . \tag{S5}$$

(d) Total work done <u>by</u> the system, from (S4) and (S5):

$$W = -(W_{12}+W_{23}+W_{34}+W_{41}) = (4s\tau_h^3/3)(\tau_h-\tau_\ell)(V_2-V_1) \quad .$$

Divided by Q_h from (S3):

$$\eta = W/Q_h = (\tau_h-\tau_\ell)/\tau_h = \eta_C \quad .$$

8.5) <u>THERMAL POLLUTION</u>

Let η be the energy conversion efficiency defined by (11), not necessarily equal to the Carnot efficiency. From $W = \eta Q_h = \eta \times (W+Q_\ell)$ we obtain

$$W = [\eta/(1-\eta)]Q_\ell \quad . \tag{S1}$$

For reversible operation, $\eta = \eta_C = (\tau_h-\tau_\ell)/\tau_h$; then

$$W = [(\tau_h-\tau_\ell)/\tau_\ell]Q_\ell \quad . \tag{S2}$$

Numerically, for our example:

$$T_h = 500°C: W = (480/293) \times 1500 \text{ MW} \cong 2460 \text{ MW} \quad ,$$

$$T_h = 600°C: W = (580/298) \times 1500 \text{ MW} \cong 2970 \text{ MW} \quad .$$

Because of unavoidable irreversibilities, the actual capacity will be less, not only because for a given amount of Q_h less power will be generated, but Q_h must actually be reduced to keep Q_ℓ fixed.

8.7) <u>LIGHT BULB IN A REFRIGERATOR</u>

Yes: We have $Q_\ell = W$. If this is inserted into (12) one obtains $T_h-T_\ell = T_\ell$ or $T_\ell = T_h/2 \cong 150$ K.