Homework #9, problem 9, 10, 13, 16, 17, 18, 24 in chapter 8

8-9 $L = [l(l+1)]^{1/2}\,\hbar$

$$4.714\times10^{-34}\text{ Js} = [l(l+1)]^{1/2}\left(\frac{6.63\times10^{-34}\text{ Js}}{2\pi}\right)$$

$$l(l+1) = \frac{\left(4.714\times10^{-34}\right)^2(2\pi)^2}{\left(6.63\times10^{-34}\right)^2} = 1.996\times10^1 \approx 20 = 4(4+1)$$

so $l = 4$.

8-10 $n = 4$, $l = 3$, and $m_l = 3$.

(a) $L = [l(l+1)]^{1/2}\,\hbar = [3(3+1)]^{1/2}\,\hbar = 2\sqrt{3}\hbar = 3.65\times10^{-34}$ Js

(b) $L_z = m_l\hbar = 3\hbar = 3.16\times10^{-34}$ Js

8-13 $Z = 2$ for He^+

(a) For $n = 3$, l can have the values of 0, 1, 2

$$\begin{aligned} l = 0 &\rightarrow m_l = 0 \\ l = 1 &\rightarrow m_l = -1,\ 0,\ +1 \\ l = 2 &\rightarrow m_l = -2,\ -1,\ 0,\ +1,\ +2 \end{aligned}$$

(b) All states have energy $E_3 = \dfrac{-Z^2}{3^2}(13.6\text{ eV})$

$$E_3 = -6.04\text{ eV}.$$

8-16 For a d state, $l = 2$. Thus, m_l can take on values –2, –1, 0, 1, 2. Since $L_z = m_l\hbar$, L_z can be $\pm2\hbar$, $\pm\hbar$, and zero.

8-17 (a) For a d state, $l = 2$

$$L = [l(l+1)]^{1/2}\,\hbar = (6)^{1/2}\left(1.055\times10^{-34}\text{ Js}\right) = 2.58\times10^{-34}\text{ Js}$$

(b) For an f state, $l = 3$

$$L = [l(l+1)]^{1/2}\,\hbar = (12)^{1/2}\left(1.055\times10^{-34}\text{ Js}\right) = 3.65\times10^{-34}\text{ Js}$$

8-18 The state is $6g$

(a) $n = 6$

(b) $E_n = -\dfrac{13.6 \text{ eV}}{n^2}$ $E_6 = \dfrac{-13.6}{6^2} \text{ eV} = -0.378 \text{ eV}$

(c) For a g-state, $l = 4$

$$L = [l(l+1)]^{1/2}\,\hbar = (4\times 5)^{1/2}\,\hbar = \sqrt{20}\hbar = 4.47\hbar$$

(d) m_l can be –4, –3, –2, –1, 0, 1, 2, 3, or 4

$L_z = m_l\hbar$; $\cos\theta = \dfrac{L_z}{L} = \dfrac{m_l}{[l(l+1)]^{1/2}}\hbar = \dfrac{m_l}{\sqrt{20}}$

m_l	–4	–3	–2	–1	0	1	2	3	4
L_z	$-4\hbar$	$-3\hbar$	$-2\hbar$	$-\hbar$	0	$\hbar$	$2\hbar$	$3\hbar$	$4\hbar$
θ	153.4°	132.1°	116.6°	102.9°	90°	77.1°	63.4°	47.9°	26.6°

8-24 $P_{1s}(r) = \dfrac{4}{a_0^3} r^2 e^{-2r/a_0}$ for hydrogen ground state, $U(r) = -\dfrac{ke^2}{r}$ is potential energy ($Z = 1$)

$$\langle U\rangle = \int_0^\infty U(r)P_{1s}(r)\,dr = -\frac{4ke^2}{a_0^3}\int_0^\infty re^{-2r/a_0}\,dr$$

$$= -\frac{4ke^2}{a_0^3}\left(\frac{a_0}{2}\right)^2\int_0^\infty ze^{-z}\,dz \quad \text{where } z = \frac{2r}{a_0}$$

$$= \frac{-ke^2}{a_0} = -2(13.6 \text{ eV}) = -27.2 \text{ eV}.$$

To find $\langle K\rangle$, we note that $\langle K\rangle + \langle U\rangle = \langle E\rangle = -\dfrac{ke^2}{2a_0} = -13.6 \text{ eV}$ so, $\langle K\rangle = \dfrac{ke^2}{a_0} = +13.6 \text{ eV}$.