Homework #8, problem 3, 16 in chapter 7 and 1, 3, 4 in chapter 8

7-3 With $E = 25\text{ MeV}$ and $U = 20\text{ MeV}$, the ratio of wavenumber is

$\frac{k_1}{k_2} = \left(\frac{E}{E-U}\right)^{1/2} = \left(\frac{25}{25-20}\right)^{1/2} = \sqrt{5} = 2.236$. Then from Problem 7-2 $R = \frac{(\sqrt{5}-1)^2}{(\sqrt{5}+1)^2} = 0.146$ and

$T = 1 - R = 0.854$. Thus, 14.6% of the incoming particles would be reflected and 85.4% would be transmitted. For electrons with the same energy, the transparency and reflectivity of the step are unchanged.

7-16 Since the alpha particle has the combined mass of 2 protons and 2 neutrons, or about $3\,755.8\ \text{MeV}/c^2$, the first approximation to the decay length δ is

$$\delta \approx \frac{\hbar}{(2mU)^{1/2}} = \frac{197.3\ \text{MeV fm}/c}{\left[2\left(3\,755.8\ \text{MeV}/c^2\right)(30\ \text{MeV})\right]^{1/2}} = 0.415\,6\ \text{fm}.$$

This gives an effective width for the (infinite) well of $R + \delta = 9.415\,6\ \text{fm}$, and a ground state energy

$E_1 = \frac{\pi^2 (197.3\ \text{MeV fm}/c)^2}{2\left(3\,755.8\ \text{MeV}/c^2\right)(9.415\,6\ \text{fm})^2} = 0.577\ \text{MeV}$. From this E we calculate $U - E = 29.42\ \text{MeV}$ and a new decay length

$$\delta = \frac{197.3\ \text{MeV fm}/c}{\left[2\left(3\,755.8\ \text{MeV}/c^2\right)(29.42\ \text{MeV})\right]^{1/2}} = 0.419\,7\ \text{fm}.$$

This, in turn, increases the effective well width to 9.419 7 fm and lowers the ground state energy to $E_1 = 0.576\ \text{MeV}$. Since our estimate for E has changed by only 0.001 MeV, we may be content with this value. With a kinetic energy of E_1, the alpha particle in the ground state has speed

$v_1 = \left(\frac{2E_1}{m}\right)^{1/2} = \left[\frac{2(0.576\ \text{MeV})}{\left(3\,755.8\ \text{MeV}/c^2\right)}\right]^{1/2} = 0.017\,5c$. In order to be ejected with a kinetic energy of 4.05 MeV, the alpha particle must have been preformed in an excited state of the nuclear well, not the ground state.

8-1 $$E = \frac{\hbar^2\pi^2}{2m}\left[\left(\frac{n_1}{L_x}\right)^2 + \left(\frac{n_2}{L_y}\right)^2 + \left(\frac{n_3}{L_z}\right)^2\right]$$

$L_x = L$, $L_y = L_z = 2L$. Let $\frac{\hbar^2\pi^2}{8mL^2} = E_0$. Then $E = E_0\left(4n_1^2 + n_2^2 + n_3^2\right)$. Choose the quantum numbers as follows:

n_1	n_2	n_3	$\frac{E}{E_0}$		
1	1	1	6		ground state
1	2	1	9	*	first two excited states
1	1	2	9	*	
2	1	1	18		

1	2	2	12	*	next excited state
2	1	2	21		
2	2	1	21		
2	2	2	24		
1	1	3	14	*	next two excited states
1	3	1	14	*	

Therefore the first 6 states are ψ_{111}, ψ_{121}, ψ_{112}, ψ_{122}, ψ_{113}, and ψ_{131} with relative energies $\frac{E}{E_0} = 6$, 9, 9, 12, 14, 14. First and third excited states are doubly degenerate.

8-3 $n^2 = 11$

(a) $$E = \left(\frac{\hbar^2\pi^2}{2mL^2}\right)n^2 = \frac{11}{2}\left(\frac{\hbar^2\pi^2}{mL^2}\right)$$

(b)

n_1	n_2	n_3	
1	1	3	
1	3	1	3-fold degenerate
3	1	1	

(c) $$\psi_{113} = A\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{\pi y}{L}\right)\sin\left(\frac{3\pi z}{L}\right)$$
$$\psi_{131} = A\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{3\pi y}{L}\right)\sin\left(\frac{\pi z}{L}\right)$$
$$\psi_{311} = A\sin\left(\frac{3\pi x}{L}\right)\sin\left(\frac{\pi y}{L}\right)\sin\left(\frac{\pi z}{L}\right)$$

8-4 (a) $\psi(x, y) = \psi_1(x)\psi_2(y)$. In the two-dimensional case, $\psi = A(\sin k_1 x)(\sin k_2 y)$ where $k_1 = \frac{n_1\pi}{L}$ and $k_2 = \frac{n_2\pi}{L}$.

(b) $$E = \frac{\hbar^2\pi^2\left(n_1^2 + n_2^2\right)}{2mL^2}$$

If we let $E_0 = \frac{\hbar^2\pi^2}{mL^2}$, then the energy levels are:

n_1	n_2	$\frac{E}{E_0}$			
1	1	1	$\rightarrow$	ψ_{11}	
1	2	$\frac{5}{2}$	$\rightarrow$	ψ_{12}	doubly degenerate
2	1	$\frac{5}{2}$	$\rightarrow$	ψ_{21}	
2	2	4	$\rightarrow$	ψ_{22}	