Homework #6, Problems 13, 33 in chapter 5; and 1, 3, 9, 13, 17 in chapter 6

5-13 A canceling wave will be produced when the path length difference between the surface reflection and the reflection from the nth plane below the surface equals some whole number of wavelengths plus $\frac{\lambda}{2}$ As the path length difference between a surface reflection and a reflection from plane n is given by $(n)(1.01\lambda)$, we find that a reflection from the 50th plane has a path difference of 50.5λ with the surface reflection, and consequently cancels the surface reflection. Essentially all waves reflected at θ will cancel as the wave reflected from the second plane will be cancelled by a reflection from the 51st plane and so on.

5-33 From the uncertainty principle, $\Delta E \Delta t \sim \hbar$ $\Delta mc^2 \Delta t = \hbar$. Therefore,

$$\frac{\Delta m}{m} = \frac{h}{2\pi c^2 \Delta t m} = \frac{h}{2\pi \Delta t E_{\text{rest}}} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi\left(8.7\times10^{-17}\ \text{s}\right)\left(135\times10^{6}\ \text{eV}\right)\left(1.6\times10^{-19}\ \text{J/eV}\right)} = 5.62\times10^{-8}.$$

6-1 (a) Not acceptable – diverges as $x \to \infty$.

(b) Acceptable.

(c) Acceptable.

(d) Not acceptable – not a single-valued function.

(e) Not acceptable – the wave is discontinuous (as is the slope).

6-3 (a) $A\sin\left(\frac{2\pi x}{\lambda}\right) = A\sin\left(5\times10^{10}x\right)$ so $\left(\frac{2\pi}{\lambda}\right) = 5\times10^{10}\ \text{m}^{-1}$, $\lambda = \frac{2\pi}{5\times10^{10}} = 1.26\times10^{-10}\ \text{m}$.

(b) $$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\ \text{Js}}{1.26\times10^{-10}\ \text{m}} = 5.26\times10^{-24}\ \text{kg m/s}$$

(c) $$K = \frac{p^2}{2m} \quad m = 9.11\times10^{-31}\ \text{kg}$$

$$K = \frac{\left(5.26\times10^{-24}\ \text{kg m/s}\right)^2}{\left(2\times9.11\times10^{-31}\ \text{kg}\right)} = 1.52\times10^{-17}\ \text{J}$$

$$K = \frac{1.52\times10^{-17}\ \text{J}}{1.6\times10^{-19}\ \text{J/eV}} = 95\ \text{eV}$$

6-9 $E_n = \frac{n^2h^2}{8mL^2}$, so $\Delta E = E_2 - E_1 = \frac{3h^2}{8mL^2}$

$$\Delta E = (3)\frac{(1240\ \text{eV nm}/c)^2}{8\left(938.28\times10^{6}\ \text{eV}/c^2\right)\left(10^{-5}\ \text{nm}\right)^2} = 6.14\ \text{MeV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV nm}}{6.14\times10^{6} \text{ eV}} = 2.02\times10^{-4} \text{ nm}$$

This is the gamma ray region of the electromagnetic spectrum.

6-13 (a) Proton in a box of width $L = 0.200 \text{ nm} = 2\times10^{-10} \text{ m}$

$$E_1 = \frac{h^2}{8m_pL^2} = \frac{\left(6.626\times10^{-34} \text{ J}\cdot\text{s}\right)^2}{8\left(1.67\times10^{-27} \text{ kg}\right)\left(2\times10^{-10} \text{ m}\right)^2} = 8.22\times10^{-22} \text{ J}$$

$$= \frac{8.22\times10^{-22} \text{ J}}{1.60\times10^{-19} \text{ J/eV}} = 5.13\times10^{-3} \text{ eV}$$

(b) Electron in the same box:

$$E_1 = \frac{h^2}{8m_eL^2} = \frac{\left(6.626\times10^{-34} \text{ J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31} \text{ kg}\right)\left(2\times10^{-10} \text{ m}\right)^2} = 1.506\times10^{-18} \text{ J} = 9.40 \text{ eV}.$$

(c) The electron has a much higher energy because it is much less massive.

6-17 (a) The wavefunctions and probability densities are the same as those shown in the two lower curves in Figure 6.16 of the text.

(b) $$P_1 = \int_{1.5\text{ Å}}^{3.5\text{ Å}} |\psi|^2\, dx = \frac{2}{10\text{ Å}}\int_{1.5\text{ Å}}^{3.5\text{ Å}} \sin^2\left(\frac{\pi x}{10}\right)dx$$

$$\frac{1}{5}\left[\frac{x}{2} - \frac{10}{4\pi}\sin\left(\frac{\pi x}{5}\right)\right]_{1.5}^{3.5}$$

In the above result we used $\int \sin^2 ax dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax)$. Therefore,

$$P_1 = \frac{1}{10}\left[x - \frac{5}{\pi}\sin\left(\frac{\pi x}{5}\right)\right]_{1.5}^{3.5} = \frac{1}{10}\left\{3.5 - \frac{5}{\pi}\sin\left[\frac{\pi(3.5)}{5}\right] - 1.5 + \frac{5}{\pi}\sin\left[\frac{\pi(1.5)}{5}\right]\right\}$$

$$= \frac{1}{10}\left[2.0 + \frac{5}{\pi}(\sin 0.3\pi - \sin 0.7\pi)\right] = \frac{1}{10}[2.00 + 0.0] = 0.200$$

(c) $$P_2 = \frac{1}{5}\int_{1.5}^{3.5}\sin^2\left(\frac{\pi x}{5}\right)dx = \frac{1}{5}\left[\frac{x}{2} - \frac{5}{4\pi}\sin(0.4\pi x)\right]_{1.5}^{3.5} = \frac{1}{10}\left[x - \frac{5}{2\pi}\sin(0.4\pi x)\right]_{1.5}^{3.5}$$

$$= \frac{1}{10}\{2.0 + (0.798)\{\sin[0.4\pi(1.5)] - \sin[0.4\pi(3.5)]\}\} = 0.351$$

(d) Using $E = \frac{n^2h^2}{8mL^2}$ we find $E_1 = 0.377 \text{ eV}$ and $E_2 = 1.51 \text{ eV}$.