Homework #5, problems 17, 18, 23, 26, 42, 44 in chapter 4

4-17 $r=\frac{n^2\hbar^2}{Zm_eke^2}=\left(\frac{n^2}{Z}\right)\left(\frac{\hbar^2}{m_eke^2}\right)$; $n=1$

$$r=\frac{1}{Z}\left[\frac{\left(1.055\times10^{-34}\ \text{Js}\right)^2}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(9\times10^{9}\ \text{Nm}^2/\text{C}^2\right)\left(1.6\times10^{-19}\ \text{C}\right)^2}\right]=\frac{5.30\times10^{-11}\ \text{m}}{Z}$$

(a) For He^+, $Z=2$, $r=\frac{5.30\times10^{-11}\ \text{m}}{2}=2.65\times10^{-11}\ \text{m}=0.026\,5\ \text{nm}$

(b) For Li^{2+}, $Z=3$, $r=\frac{5.30\times10^{-11}\ \text{m}}{3}=1.77\times10^{-11}\ \text{m}=0.017\,7\ \text{nm}$

(c) For Be^{3+}, $Z=4$, $r=\frac{5.30\times10^{-11}\ \text{m}}{4}=1.32\times10^{-11}\ \text{m}=0.013\,2\ \text{nm}$

4-18 (a) $\Delta E=(13.6\ \text{eV})\left(\frac{1}{1^2}-\frac{1}{3^2}\right)=12.1\ \text{eV}$

(b) Either $\Delta E=12.1\ \text{eV}$ or $\Delta E=(13.6\ \text{eV})\left(\frac{1}{1}-\frac{1}{2^2}\right)=10.2\ \text{eV}$ and

$\Delta E=(13.6\ \text{eV})\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=1.89\ \text{eV}$.

4-23 (a) $r_1=\left(0.052\,9\ \text{nm}\right)n^2=0.052\,9\ \text{nm}$ (when $n=1$)

(b) $m_ev=m_e\left(\frac{ke^2}{m_er}\right)^{1/2}$

$$m_e=\left[\frac{\left(9.1\times10^{-31}\ \text{kg}\right)\left(9\times10^{9}\ \text{Nm}^2/\text{C}^2\right)}{5.29\times10^{-11}\ \text{m}}\right]^{1/2}\times\left(1.6\times10^{-19}\ \text{C}\right)$$

$M_ev=1.99\times10^{-24}\ \text{kg m/s}$

(c) $L=m_evr=\left(1.99\times10^{-24}\ \text{kg m/s}\right)\left(5.29\times10^{-11}\ \text{m}\right)$, $L=1.05\times10^{-34}\left(\text{kg m}^2/\text{s}\right)=\hbar$

(d) $K=|E|=13.6\ \text{eV}$

(e) $U=-2K=-27.2\ \text{eV}$

(f) $E=K+U=-13.6\ \text{eV}$

4-26 Lyman series has $n_\text{f}=1$, λ_max has $n_\text{i}=2$; λ_min has $n_\text{i}=\infty$

$$\frac{1}{\lambda_{\max}} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = R\left(\frac{1-1}{2^2}\right) = \frac{3R}{4}$$
$$\lambda_{\max} = \frac{4}{3R} = \frac{4}{(3)\left(1.097\times 10^7\ \text{m}^{-1}\right)} = 121.5\ \text{nm}$$
$$\lambda_{\min} = \frac{1}{R} = 91.16\ \text{nm}$$

As shown on the inside front cover, the visible spectrum begins at about 350 nm, so the Lyman series is in the UV.

4-42 Each atom gives up its kinetic energy in emitting a photon.

$$\frac{1}{2}m_p v^2 = \frac{hc}{\lambda} = \frac{1.24\times 10^3 eV \bullet nm}{121.6nm} = 10.2eV$$
$$v = 4.4\times 10^4 m/s$$

4-44 (a) $\frac{1}{\lambda} = Z^2 R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$. The shortest wavelength, λ_s, corresponds to $n_i = \infty$, and the longest wavelength, λ_ℓ, to $n_i = n_f + 1$.

$$\frac{1}{\lambda_s} = \frac{Z^2 R_H}{n_f^2} \qquad (1)$$

$$\frac{1}{\lambda_\ell} = Z^2 R_H\left[\frac{1}{n_f^2} - \frac{1}{\left(n_f+1\right)^2}\right] = \frac{Z^2 R_H}{n_f^2}\left[1 - \left(\frac{n_f}{n_f+1}\right)^2\right] \qquad (2)$$

Divide (1) and (2): $\frac{\lambda_s}{\lambda_\ell} = 1 - \left(\frac{n_f}{n_f+1}\right)^2$, $\therefore \frac{n_f}{n_f+1} = \sqrt{1 - \frac{\lambda_s}{\lambda_\ell}} = \sqrt{1 - \frac{22.8\ \text{nm}}{63.3\ \text{nm}}} = 0.800$,

$\therefore n_f = 4$. From (1): $Z = \sqrt{\frac{n_f^2}{\lambda_s R_H}} = \sqrt{\frac{4^2}{\left(22.8\times 10^{-9}\ \text{m}\right)\left(1.097\times 10^7\ \text{m}^{-1}\right)}} = 8.00$. Hence the ion is O^{7+}.

(b) $\lambda = \left\{\left(7.020\,8\times 10^8\ \text{m}^{-1}\right)\left[\frac{1}{4^2} - \frac{1}{(4+k)^2}\right]\right\}^{-1}$, $k = 1,\ 2,\ 3,\ \ldots$. Setting $k = 2,\ 3,\ 4$ gives $\lambda = 41.0\ \text{nm}$, 33.8 nm, 30.4 nm