Homework #4, problem 31, 35, 37 in chapter and 3, 17, 23, 26 in chapter 4

3-31 (a)

$$E' = \frac{hc}{\lambda'},\ \lambda' = \lambda_0 + \Delta\lambda$$

$$\lambda_0 = \frac{hc}{E_0} = \frac{(6.63\times10^{-34}\ \mathrm{J\cdot s})(3\times10^{8}\ \mathrm{m/s})}{0.1\ \mathrm{MeV}} = 1.243\times10^{-11}\ \mathrm{m}$$

$$\Delta\lambda = \left(\frac{h}{m_e c}\right)(1-\cos\theta) = \frac{(6.63\times10^{-34}\ \mathrm{J\cdot s})(1-\cos 60°)}{(9.11\times10^{-34}\ \mathrm{kg})(3\times10^{8}\ \mathrm{m/s})} = 1.215\times10^{-12}\ \mathrm{m}$$

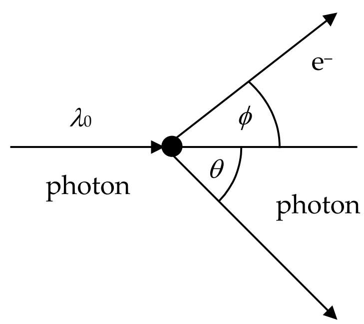
$$\lambda' = \lambda_0 + \Delta\lambda = 1.364\times10^{-11}\ \mathrm{m}$$

$$E' = \frac{hc}{\lambda'} = \frac{(6.63\times10^{-34}\ \mathrm{J\cdot s})(3\times10^{8}\ \mathrm{m/s})}{1.364\times10^{-11}\ \mathrm{m}} = 9.11\times10^{4}\ \mathrm{eV}$$

(b)

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + K_e$$

$$K_e = 0.1\ \mathrm{MeV} - 91.1\ \mathrm{keV} = 8.90\ \mathrm{keV}$$



(c) Conservation of momentum along x: $\frac{h}{\lambda_0} = \left(\frac{h}{\lambda'}\right)\cos\theta + \gamma m_e v\cos\phi$. Conservation of momentum along y: $\left(\frac{h}{\lambda'}\right)\sin\theta = \gamma m_e v \sin\phi$. So that

$$\frac{\gamma m_e v \sin\phi}{\gamma m_e v\cos\phi} = \left(\frac{h}{\lambda'}\right)\sin\theta\left[\left(\frac{h}{\lambda_0}\right) - \left(\frac{h}{\lambda'}\right)\cos\theta\right]$$

$$\tan\phi = \frac{\lambda_0 \sin\theta}{(\lambda' - \lambda_0)\cos\theta}$$

Here, $\theta = 60°$, $\lambda_0 = 1.243\times10^{-11}\ \mathrm{m}$, and $\lambda' = 1.364\times10^{-11}\ \mathrm{m}$. Consequently,

$$\tan\phi = \frac{(1.24\times10^{-11}\ \mathrm{m})(\sin 60°)}{(1.36-1.24\cos 60°)\times10^{-11}\ \mathrm{m}} = 1.451$$

$$\phi = 55.4°$$

3-35 (a) The energy vs wavelength relation for a photon is $E=\frac{hc}{\lambda}$. For a photon of wavelength given by $\lambda_0 = 0.071\,1\text{ nm}$ the photon's energy is

$$E=\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3\times10^{8}\text{ m/s}\right)}{\left(0.071\,1\times10^{-9}\text{ m}\right)\left(1.602\times10^{-19}\text{ J/eV}\right)}=17.4\text{ keV}$$

(b) For the case of back scattering, $\theta=\pi$ the Compton scattering relation becomes $\lambda'-\lambda_0=\left(\frac{2hc}{m_e c^2}\right)$. Setting $\lambda_0 = 0.071\,1\text{ nm}$ we obtain

$$\lambda' = 0.711\text{ nm}+\frac{2hc}{m_e c^2}=7.60\times10^{-11}$$

or 0.076 0 nm.

(c) $$E'=\frac{hc}{\lambda'}=\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3\times10^{8}\text{ m/s}\right)}{\left(7.60\times10^{-11}\text{ m}\right)\left(1.602\times10^{-19}\text{ J/eV}\right)}=16.3\text{ keV}.$$

(d) $\Delta E = 17.45\text{ keV}-16.33\text{ keV}=1.12\text{ keV}\sim1.1\text{ keV}$.

3-37 When waves are scattered between two adjacent planes of a single crystal, constructive wave interference will occur when the path length difference between such reflected waves is an integer multiple of wavelengths. This condition is expressed by the Bragg equation for constructive interference, $2d\sin\theta=n\lambda$ where d is the distance between adjacent crystalline planes, θ is the angle of incidence of the x-ray beam of photons, n is an integer for constructive interference, and λ is the wavelength of the photon beam which is in this case, 0.062 6 nm. Ignoring the incident beam that is not scattered, the first three angles for which maxima of x-ray intensities are found are $1\lambda=2d\sin\theta_1$ or

$$\sin\theta_1=\frac{\lambda}{2d}=\frac{0.626\times10^{-10}\text{ m}}{8\times10^{-10}\text{ m}}$$
$$\theta_1 = 0.078\,3\text{ radians}=4.49°$$

$2\lambda=2d\sin\theta_2$ or

$$\sin\theta_2=\frac{\lambda}{d}=\frac{0.626\times10^{-10}\text{ m}}{4.0\times10^{-10}\text{ m}}=0.156\,5,\ \theta=9.00°$$

$3\lambda=2d\sin\theta_3$ or

$$\sin\theta_3=\frac{3\lambda}{2d}=\frac{3\left(0.626\times10^{-10}\text{ m}\right)}{8\times10^{-10}\text{ m}}=0.234\,75,\ \theta_3=13.6°$$

4-3 Thomson's device will work for positive and negative particles, so we may apply $\frac{q}{m} \approx \frac{V\theta}{B^2 ld}$.

(a) $\frac{q}{m} \approx \frac{V\theta}{B^2 ld} = (2\,000\text{ V})\frac{0.20\text{ radians}}{\left(4.57\times10^{-2}\text{ T}\right)^2}(0.10\text{ m})(0.02\text{ m}) = 9.58\times10^{7}\text{ C/kg}$

(b) As the particle is attracted by the negative plate, it carries a positive charge and is a proton. $\left(\frac{q}{m_p} = \frac{1.60\times10^{-19}\text{ C}}{1.67\times10^{-27}\text{ kg}} = 9.58\times10^{7}\text{ C/kg}\right)$

(c) $v_x = \frac{E}{B} = \frac{V}{\text{d}B} = \frac{2\,000\text{ V}}{0.02\text{ m}}\left(4.57\times10^{-2}\text{ T}\right) = 2.19\times10^{6}\text{ m/s}$

(d) As $v_x \sim 0.01c$ there is no need for relativistic mechanics.

4-17 $r = \frac{n^2\hbar^2}{Zm_e ke^2} = \left(\frac{n^2}{Z}\right)\left(\frac{\hbar^2}{m_e ke^2}\right)$; $n = 1$

$$r = \frac{1}{Z}\left[\frac{\left(1.055\times10^{-34}\text{ Js}\right)^2}{\left(9.11\times10^{-31}\text{ kg}\right)\left(9\times10^{9}\text{ Nm}^2/\text{C}^2\right)\left(1.6\times10^{-19}\text{ C}\right)^2}\right] = \frac{5.30\times10^{-11}\text{ m}}{Z}$$

(a) For He^{+}, $Z = 2$, $r = \frac{5.30\times10^{-11}\text{ m}}{2} = 2.65\times10^{-11}\text{ m} = 0.026\,5\text{ nm}$

(b) For Li^{2+}, $Z = 3$, $r = \frac{5.30\times10^{-11}\text{ m}}{3} = 1.77\times10^{-11}\text{ m} = 0.017\,7\text{ nm}$

(c) For Be^{3+}, $Z = 4$, $r = \frac{5.30\times10^{-11}\text{ m}}{4} = 1.32\times10^{-11}\text{ m} = 0.013\,2\text{ nm}$

4-23 (a) $r_1 = (0.052\,9\text{ nm})n^2 = 0.052\,9\text{ nm}$ (when $n = 1$)

(b) $m_e v = m_e\left(\frac{ke^2}{m_e r}\right)^{1/2}$

$$m_e = \left[\frac{\left(9.1\times10^{-31}\text{ kg}\right)\left(9\times10^{9}\text{ Nm}^2/\text{C}^2\right)}{5.29\times10^{-11}\text{ m}}\right]^{1/2}\times\left(1.6\times10^{-19}\text{ C}\right)$$

$M_e v = 1.99\times10^{-24}\text{ kg m/s}$

(c) $L = m_e vr = \left(1.99\times10^{-24}\text{ kg m/s}\right)\left(5.29\times10^{-11}\text{ m}\right)$, $L = 1.05\times10^{-34}\left(\text{kg m}^2/\text{s}\right) = \hbar$

(d) $K = |E| = 13.6\text{ eV}$

(e) $U = -2K = -27.2\text{ eV}$

(f) $E = K + U = -13.6\ \text{eV}$

4-26 Lyman series has $n_\text{f} = 1$, λ_max has $n_\text{i} = 2$; λ_min has $n_\text{i} = \infty$

$$\frac{1}{\lambda_\text{max}} = R\left(\frac{1}{n_\text{f}^2} - \frac{1}{n_\text{i}^2}\right) = R\left(\frac{1-1}{2^2}\right) = \frac{3R}{4}$$

$$\lambda_\text{max} = \frac{4}{3R} = \frac{4}{(3)\left(1.097\times10^7\ \text{m}^{-1}\right)} = 121.5\ \text{nm}$$

$$\lambda_\text{min} = \frac{1}{R} = 91.16\ \text{nm}$$

As shown on the inside front cover, the visible spectrum begins at about 350 nm, so the Lyman series is in the UV.