Homework #2, problem 16, 18 in chapter 1 and problem 2, 13, 17, 21, 29 in chapter 2

1-16 For an observer approaching a light source, $\lambda_{\text{ob}} = \left[\frac{(1-v/c)^{1/2}}{(1+v/c)^{1/2}}\right]\lambda_{\text{source}}$. Setting $\beta = \frac{v}{c}$ and after some algebra we find,

$$\beta = \frac{\lambda_{\text{source}}^2 - \lambda_{\text{obs}}^2}{\lambda_{\text{source}}^2 + \lambda_{\text{obs}}^2} = \frac{(650\text{ nm})^2 - (550\text{ nm})^2}{(650\text{ nm})^2 + (550\text{ nm})^2} = 0.166$$

$$v = 0.166c = \left(4.98\times10^7\ \text{m/s}\right)(2.237\ \text{mi/h})(\text{m/s})^{-1} = 1.11\times10^8\ \text{mi/h}.$$

1-18 (a) Let f_c be the frequency as seen by the car. Thus, $f_c = f_{\text{source}}\sqrt{\frac{c+v}{c-v}}$ and, if f is the frequency of the reflected wave, $f = f_c\sqrt{\frac{c+v}{c-v}}$. Combining these equations gives $f = f_{\text{source}}\frac{(c+v)}{(c-v)}$.

(b) Using the above result, $f(c-v) = f_{\text{source}}(c+v)$, which gives

$$(f - f_{\text{source}})c = (f + f_{\text{source}})v \approx 2f_{\text{source}}v.$$

The beat frequency is then $f_{\text{beat}} = f - f_{\text{source}} = \frac{2f_{\text{source}}v}{c} = \frac{2v}{\lambda}$.

(c)
$$f_{\text{beat}} = \frac{2(30.0\ \text{m/s})\left(10.0\times10^9\ \text{Hz}\right)}{3.00\times10^8\ \text{m/s}} = \frac{2(30.0\ \text{m/s})}{0.030\,0\ \text{m}} = 2\,000\ \text{Hz} = 2.00\ \text{kHz}$$

$$\lambda = \frac{c}{f_{\text{source}}} = \frac{3.00\times10^8\ \text{m/s}}{10.0\times10^9\ \text{Hz}} = 3.00\ \text{cm}$$

(d) $v = \frac{f_{\text{beat}}\lambda}{2}$ so,

$$\Delta v = \frac{\Delta f_{\text{beat}}\lambda}{2} = \frac{(5\ \text{Hz})(0.030\,0\ \text{m})}{2} = 0.075\,0\ \text{m/s} \approx 0.2\ \text{mi/h}$$

2-2 (a) Scalar equations can be considered in this case because relativistic and classical velocities are in the same direction.

$$p = \gamma mv = 1.90mv = \frac{mv}{\left[1-(v/c)^2\right]^{1/2}} \Rightarrow \frac{1}{\left[1-(v/c)^2\right]^{1/2}} = 1.90 \Rightarrow v = \left[1-\left(\frac{1}{1.90}\right)^2\right]^{1/2}c$$

$$= 0.85c$$

(b) No change, because the masses cancel each other.

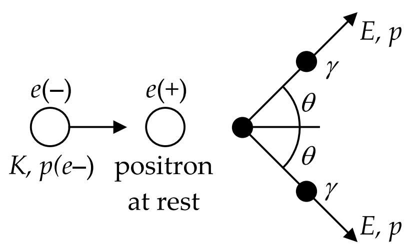
2-13 (a)
$$E = 400mc^2 = \gamma mc^2$$
$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = 400$$
$$\left(1 - \frac{v^2}{c^2}\right) = \left(\frac{1}{400}\right)^2$$
$$\frac{v}{c} = \left[1 - \frac{1}{400^2}\right]^{1/2}$$
$$v = 0.999\,997c$$

(b)
$$K = E - mc^2 = (400 - 1)mc^2 = 399mc^2 = (399)(938.3\text{ MeV}) = 3.744 \times 10^5\text{ MeV}$$

2-17 $\Delta m = m_{\text{Ra}} - m_{\text{Rn}} - m_{\text{He}}$ (an atomic unit of mass, the u, is one-twelfth the mass of the ^{12}C atom or $1.660\,54 \times 10^{-27}$ kg)

$$\Delta m = (226.025\,4 - 22.017\,5 - 4.002\,6)\text{ u} = 0.005\,3\text{ u}$$
$$E = (\Delta m)(931\text{ MeV/u}) = (0.005\,3\text{ u})(931\text{ MeV/u}) = 4.9\text{ MeV}$$

2-21



Conservation of mass-energy requires $K + 2mc^2 = 2E$ where K is the electron's kinetic energy, m is the electron's mass, and E is the gamma ray's energy.

$$E = \frac{K}{2} + mc^2 = (0.500 + 0.511)\text{ MeV} = 1.011\text{ MeV}.$$

Conservation of momentum requires that $p_{e^-} = 2p\cos\theta$ where p_{e^-} is the initial momentum of the electron and p is the gamma ray's momentum, $\frac{E}{c} = 1.011\text{ MeV}/c$. Using $E_{e^-}^2 = p_{e^-}^2 c^2 + \left(mc^2\right)^2$ where E_{e^-} is the electron's total energy, $E_{e^-} = K + mc^2$, yields

$$p_{e^-} = \frac{1}{c}\sqrt{K^2 + 2Kmc^2} = \frac{\sqrt{(1.00)^2 + 2(1.00)(0.511)}\text{ MeV}}{c} = 1.422\text{ MeV}/c.$$

Finally, $\cos\theta = \frac{p_{e^-}}{2p} = 0.703$; $\theta = 45.3°$.

2-29 The energy of the first fragment is given by $E_1^2 = p_1^2 c^2 + \left(m_1 c^2\right)^2 = (1.75\text{ MeV})^2 + (1.00\text{ MeV})^2$; $E_1 = 2.02\text{ MeV}$. For the second, $E_2^2 = (2.00\text{ MeV})^2 + (1.50\text{ MeV})^2$; $E_2 = 2.50\text{ MeV}$.

(a) Energy is conserved, so the unstable object had $E = 4.52\text{ MeV}$. Each component of momentum is conserved, so the original object moved with

$$p^2 = p_x^2 + p_y^2 = \left(\frac{1.75\text{ MeV}}{c}\right)^2 + \left(\frac{2.00\text{ MeV}}{c}\right)^2.$$

Then for $(4.52\text{ MeV})^2 = (1.75\text{ MeV})^2 + (2.00\text{ MeV})^2 + \left(mc^2\right)^2$; $m = 3.65\text{ MeV}/c^2$.

(b) Now $E = \gamma mc^2$ gives $4.52\text{ MeV} = \frac{1}{\sqrt{1 - v^2/c^2}} 3.65\text{ MeV}$; $1 - \frac{v^2}{c^2} = 0.654$ and $v = 0.589c$.