

Homework #11, problems 16 in chapter 7; and 1, 3, 4, 9, 10 in chapter 8

7-16 Since the alpha particle has the combined mass of 2 protons and 2 neutrons, or about $3755.8 \text{ MeV}/c^2$, the first approximation to the decay length δ is

$$\delta \approx \frac{\hbar}{(2mU)^{1/2}} = \frac{197.3 \text{ MeV fm}/c}{[2(3755.8 \text{ MeV}/c^2)(30 \text{ MeV})]^{1/2}} = 0.4156 \text{ fm} .$$

This gives an effective width for the (infinite) well of $R + \delta = 9.4156 \text{ fm}$, and a ground state

$$\text{energy } E_1 = \frac{\pi^2 (197.3 \text{ MeV fm}/c)^2}{2(3755.8 \text{ MeV}/c^2)(9.4156 \text{ fm})^2} = 0.577 \text{ MeV} . \text{ From this } E \text{ we calculate}$$

$U - E = 29.42 \text{ MeV}$ and a new decay length

$$\delta = \frac{197.3 \text{ MeV fm}/c}{[2(3755.8 \text{ MeV}/c^2)(29.42 \text{ MeV})]^{1/2}} = 0.4197 \text{ fm} .$$

This, in turn, increases the effective well width to 9.4197 fm and lowers the ground state energy to $E_1 = 0.576 \text{ MeV}$. Since our estimate for E has changed by only 0.001 MeV , we may be content with this value. With a kinetic energy of E_1 , the alpha particle in the ground state has speed

$$v_1 = \left(\frac{2E_1}{m} \right)^{1/2} = \left[\frac{2(0.576 \text{ MeV})}{(3755.8 \text{ MeV}/c^2)} \right]^{1/2} = 0.0175c . \text{ In order to be ejected with a kinetic energy of}$$

4.05 MeV , the alpha particle must have been preformed in an excited state of the nuclear well, not the ground state.

8-1
$$E = \frac{\hbar^2 \pi^2}{2m} \left[\left(\frac{n_1}{L_x} \right)^2 + \left(\frac{n_2}{L_y} \right)^2 + \left(\frac{n_3}{L_z} \right)^2 \right]$$

$L_x = L$, $L_y = L_z = 2L$. Let $\frac{\hbar^2 \pi^2}{8mL^2} = E_0$. Then $E = E_0 (4n_1^2 + n_2^2 + n_3^2)$. Choose the quantum numbers as follows:

n_1	n_2	n_3	$\frac{E}{E_0}$	
1	1	1	6	ground state
1	2	1	9	* first two excited states
1	1	2	9	*
2	1	1	18	
1	2	2	12	* next excited state
2	1	2	21	
2	2	1	21	
2	2	2	24	
1	1	3	14	* next two excited states
1	3	1	14	*

Therefore the first 6 states are ψ_{111} , ψ_{121} , ψ_{112} , ψ_{122} , ψ_{113} , and ψ_{131} with relative energies $\frac{E}{E_0} = 6, 9, 9, 12, 14, 14$. First and third excited states are doubly degenerate.

8-3 $n^2 = 11$

(a)
$$E = \left(\frac{\hbar^2 \pi^2}{2mL^2} \right) n^2 = \frac{11}{2} \left(\frac{\hbar^2 \pi^2}{mL^2} \right)$$

(b)

n_1	n_2	n_3	
1	1	3	
1	3	1	3-fold degenerate
3	1	1	

(c)
$$\begin{aligned} \psi_{113} &= A \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{\pi y}{L}\right) \sin\left(\frac{3\pi z}{L}\right) \\ \psi_{131} &= A \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{3\pi y}{L}\right) \sin\left(\frac{\pi z}{L}\right) \\ \psi_{311} &= A \sin\left(\frac{3\pi x}{L}\right) \sin\left(\frac{\pi y}{L}\right) \sin\left(\frac{\pi z}{L}\right) \end{aligned}$$

8-4 (a) $\psi(x, y) = \psi_1(x)\psi_2(y)$. In the two-dimensional case, $\psi = A(\sin k_1 x)(\sin k_2 y)$ where $k_1 = \frac{n_1 \pi}{L}$ and $k_2 = \frac{n_2 \pi}{L}$.

(b)
$$E = \frac{\hbar^2 \pi^2 (n_1^2 + n_2^2)}{2mL^2}$$

If we let $E_0 = \frac{\hbar^2 \pi^2}{mL^2}$, then the energy levels are:

n_1	n_2	$\frac{E}{E_0}$		
1	1	1	→	ψ_{11}
1	2	$\frac{5}{2}$	→	ψ_{12}
2	1	$\frac{5}{2}$	→	ψ_{21}
2	2	4	→	ψ_{22}

} doubly degenerate

8-9 $L = [l(l+1)]^{1/2} \hbar$

$$4.714 \times 10^{-34} \text{ Js} = [l(l+1)]^{1/2} \left(\frac{6.63 \times 10^{-34} \text{ Js}}{2\pi} \right)$$

$$l(l+1) = \frac{(4.714 \times 10^{-34})^2 (2\pi)^2}{(6.63 \times 10^{-34})^2} = 1.996 \times 10^1 \approx 20 = 4(4+1)$$

so $l = 4$.

8-10 $n = 4$, $l = 3$, and $m_l = 3$.

(a) $L = [l(l+1)]^{1/2} \hbar = [3(3+1)]^{1/2} \hbar = 2\sqrt{3}\hbar = 3.65 \times 10^{-34} \text{ Js}$

(b) $L_z = m_l \hbar = 3\hbar = 3.16 \times 10^{-34} \text{ Js}$