Homework #10, problems 4, 9, 10, 13, 17, 21 in chapter 9

9-4 (a) $3d$ subshell $\Rightarrow l = 2 \Rightarrow m_l = -2, -1, 0, 1, 2$ and $m_s = \pm\frac{1}{2}$ for each m_l

n	l	m_l	m_s
3	2	–2	–1/2
3	2	–2	+1/2
3	2	–1	–1/2
3	2	–1	+1/2
3	2	0	–1/2
3	2	0	+1/2
3	2	1	–1/2
3	2	1	+1/2
3	2	2	–1/2
3	2	2	+1/2

(b) $3p$ subshell: for a p state, $l = 1$. Thus m_l can take on values $-l$ to l, or –1, 0, 1. For each m_l, m_s can be $\pm\frac{1}{2}$.

n	l	m_l	m_s
3	1	–1	–1/2
3	1	–1	+1/2
3	1	0	–1/2
3	1	0	+1/2
3	1	1	–1/2
3	1	1	+1/2

9-9 With $s = \frac{3}{2}$, the spin magnitude is $|\mathbf{S}| = [s(s+1)]^{1/2}\,\hbar = \left(\frac{[15]^{1/2}}{2}\right)\hbar$. The z-component of spin is $S_z = m_s\hbar$ where m_s ranges from $-s$ to s in integer steps or, in this case, $m_s = -\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2}$. The spin vector S is inclined to the z-axis by an angle θ such that

$$\cos(\theta) = \frac{S_z}{|\mathbf{S}|} = \frac{m_s\hbar}{\left([15]^{1/2}/2\right)\hbar} = \frac{m_s}{[15]^{1/2}/2} = -\frac{3}{(15)^{1/2}},\ -\frac{1}{(15)^{1/2}},\ +\frac{1}{(15)^{1/2}},\ +\frac{3}{(15)^{1/2}}$$

or $\theta = 140.8°, 105.0°, 75.0°, 39.2°$. The Ω^- *does* obey the Pauli Exclusion Principle, since the spin s of this particle is half-integral, as it is for all fermions.

9-10 $n = 2$; $l = 1, 0$; $s = \frac{1}{2}$

$j = 1+\frac{1}{2},\ 1-\frac{1}{2},\ 0+\frac{1}{2},\ \left|0-\frac{1}{2}\right|$

For $j = \frac{3}{2}$; $m_j = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$

For $j=\frac{1}{2};\ m_j=-\frac{1}{2},\ \frac{1}{2}$

9-13 (a) $4F_{5/2}\rightarrow n=4,\ l=3,\ j=\frac{5}{2}$

(b) $|\mathbf{J}|=[j(j+1)]^{1/2}\hbar=\left[\left(\frac{5}{2}\right)\left(\frac{7}{2}\right)\right]^{1/2}\hbar=\left[\frac{35}{4}\right]^{1/2}\hbar=\left[\frac{(35)^{1/2}}{2}\right]\hbar$

(c) $J_z=m_j\hbar$ where m_j can be $-j,\ -j+1,\ \ldots,\ j-1,\ j$ so here m_j can be $-\frac{5}{2},\ -\frac{3}{2},\ -\frac{1}{2},\ \frac{1}{2},\ \frac{3}{2},\ \frac{5}{2}$. J_z can be $-\frac{5}{2}\hbar,\ -\frac{3}{2}\hbar,\ -\frac{1}{2}\hbar,\ \frac{1}{2}\hbar,\ \frac{3}{2}\hbar$, or $\frac{5}{2}\hbar$.

9-17 From Equation 8.9 we have $E=\left(\frac{\hbar^2\pi^2}{2mL^2}\right)\left(n_1^2+n_2^2+n_3^2\right)$

$$E=\frac{\left(1.054\times10^{-34}\right)^2\left(\pi^2\right)\left(n_1^2+n_2^2+n_3^2\right)}{2\left(9.11\times10^{-31}\right)\left(2\times10^{-10}\right)^2}=\left(1.5\times10^{-18}\text{ J}\right)\left(n_1^2+n_2^2+n_3^2\right)=(9.4\text{ eV})\left(n_1^2+n_2^2+n_3^2\right)$$

(a) 2 electrons per state. The lowest states have

$$\left(n_1^2+n_2^2+n_3^2\right)=(1,\ 1,\ 1)\Rightarrow E_{111}=(9.4\text{ eV})\left(1^2+1^2+1^2\right)\text{ eV}=28.2\text{ eV}.$$

For $\left(n_1^2+n_2^2+n_3^2\right)=(1,\ 1,\ 2)$ or $(1,\ 2,\ 1)$ or (2, 1, 1),

$$E_{112}=E_{121}=E_{211}=(9.4\text{ eV})\left(1^2+1^2+2^2\right)=56.4\text{ eV}$$
$$E_{\min}=2\times(E_{111}+E_{112}+E_{121}+E_{211})=2(28.2+3\times56.4)=398.4\text{ eV}$$

(b) All 8 particles go into the $\left(n_1^2+n_2^2+n_3^2\right)=(1,\ 1,\ 1)$ state, so

$$E_{\min}=8\times E_{111}=225.6\text{ eV}.$$

9-21 (a) $1s^2 2s^2 2p^4$

(b) For the two 1s electrons, $n=1$, $l=0$, $m_l=0$, $m_s=\pm\frac{1}{2}$.

For the two 2s electrons, $n=2$, $l=0$, $m_l=0$, $m_s=\pm\frac{1}{2}$.

For the four 2p electrons, $n=2$, $l=1$, $m_l=1,\ 0,\ -1$, $m_s=\pm\frac{1}{2}$.