Homework #1, Problems: 8, 10, 12, 13, 21, 23, 24 in chapter 1

1-8 $L = \dfrac{L'}{\gamma}$

$$\frac{1}{\gamma} = \frac{L}{L'} = \left[1 - \frac{v^2}{c^2}\right]^{1/2}$$

$$v = c\left[1 - \left(\frac{L}{L'}\right)^2\right]^{1/2} = c\left[1 - \left(\frac{75}{100}\right)^2\right]^{1/2} = 0.661c$$

1-10 (a) $\tau = \gamma\tau'$ where $\beta = \dfrac{v}{c}$ and

$$\gamma =' \left(1 - \beta^2\right)^{-1/2} = \tau'\left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \left(2.6\times10^{-8}\ \text{s}\right)\left[1 - (0.95)^2\right]^{-1/2} = 8.33\times10^{-8}\ \text{s}$$

(b) $d = v\tau = (0.95)\left(3\times10^8\right)\left(8.33\times10^8\ \text{s}\right) = 24\ \text{m}$

1-12 (a) 70 beats/min or $\Delta t' = \dfrac{1}{70}$ min

(b) $\Delta t = \gamma\,\Delta t' = \left[1 - (0.9)^2\right]^{-1/2}\left(\dfrac{1}{70}\right)\text{min} = 0.032\,8$ min/beat or the number of beats per minute $\approx 30.5 \approx 31$.

1-13 (a) $\tau = \gamma\tau' = \left[1 - (0.95)^2\right]^{-1/2}(2.2\ \mu\text{s}) = 7.05\ \mu\text{s}$

(b) $\Delta t' = \dfrac{d}{0.95c} = \dfrac{3\times10^3\ \text{m}}{0.95c} = 1.05\times10^{-5}$ s, therefore,

$$N = N_0\exp\left(-\frac{\Delta t}{\tau}\right) = \left(5\times10^4\ \text{muons}\right)\exp(-1.487) \approx 1.128\times10^4\ \text{muons}.$$

1-21 $u'_X = \dfrac{u_X - v}{1 - u_X v/c^2} = \dfrac{0.50c - 0.80c}{1 - (0.50c)(0.80c)/c^2} = -0.50c$

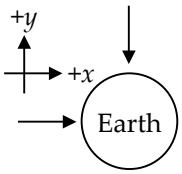
1-23 (a) Let event 1 have coordinates $x_1 = y_1 = z_1 = t_1 = 0$ and event 2 have coordinates $x_2 = 100$ mm, $y_2 = z_2 = t_2 = 0$. In S', $x'_1 = \gamma(x_1 - vt_1) = 0$, $y'_1 = y_1 = 0$, $z'_1 = z_1 = 0$, and $t'_1 = \gamma\left[t_1 - \left(\dfrac{v}{c^2}\right)x_1\right] = 0$, with $\gamma = \left[1 - \dfrac{v^2}{c^2}\right]^{-1/2}$ and so $\gamma = \left[1 - (0.70)^2\right]^{-1/2} = 1.40$. In system S', $x'_2 = \gamma(x_2 - vt_2) = 140$ m, $y'_2 = z'_2 = 0$, and

$$t_2' = \gamma\left[t_2 - \left(\frac{v}{c^2}\right)x_2\right] = \frac{(1.4)(-0.70)(100\text{ m})}{3.00\times10^8\text{ m/s}} = -0.33\ \mu\text{s}.$$

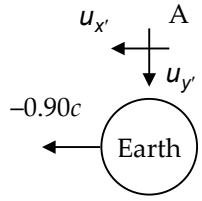
(b) $\Delta x' = x_2' - x_1' = 140$ m

(c) Events are not simultaneous in S', event 2 occurs $0.33\ \mu$s earlier than event 1.

1-24 A. $u_y = -0.90c$



B. $u_x = -0.90c$



B. at rest

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2} = \frac{0 - 0.90c}{1 - (0)(0.90c)/c^2} = -0.90c$$

$$u_y' = \frac{u_y}{\gamma\left(1 - u_x v/c^2\right)} = \frac{0 - 0.90c}{[1 - 0.81]^{-1/2}} \cong -0.392c$$

The speed of A as measured by B is

$$u_{AB} = \left[(u_x')^2 + \left(u_y'\right)^2\right]^{1/2} = \left[(-0.90c)^2 + (-0.392c)^2\right]^{1/2} = 0.982c.$$

Classically, $u_{AB} = 1.3c$.