PHY 335
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Planck's Constant

History of Planck's Constant

Planck's constant, represented by ***h***, is a constant that was derived towards the end of the 19th century by Max Planck. Planck was interested in the universal character of the distribution law which was developed by Kirchhoff's theorem. Kirchhoff had shown the nature of radiation in a thermal equilibrium in an enclosure, where the walls are fixed at a certain temperature, is completely independent of the properties of any material bodies, which are in equilibrium with the radiation (Boyer). Planck pursued his constant due to a missing explanation for the shape of the emission spectrum of black-body radiation. Knowing that everything emits some amount of electromagnetic radiation, Planck found that low frequencies (i.e. long wavelengths) tended towards Rayleigh-Jean's law and high frequencies (i.e. short wavelengths) tended towards the Wien approximation.

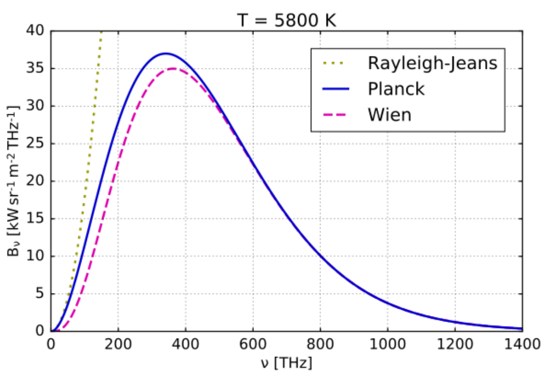


Fig 1. (Left) This represents a comparison between Rayleigh-Jean's Law, Planck's Law, and the Wien approximation.

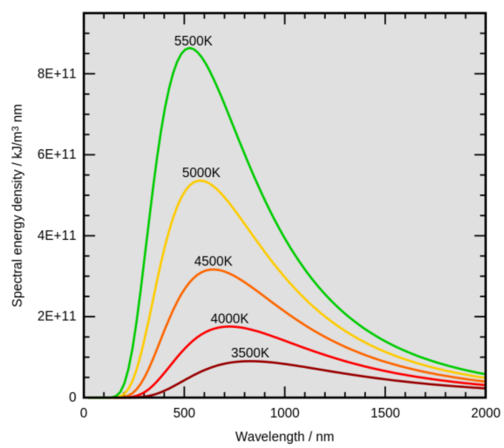


Fig 2. (Right) This represents Wien's Law of radiation, showing the intensity of light emitted from a black body at certain wavelengths. Each curve represents a different temperature.

Planck studied how these curves represented that of a set of harmonic oscillators, a separate one per frequency, and noted that their entropy varied with the temperature in an attempt to correlate with Wien's Law. From this, Planck was able to approximate a derivation of a function to represent the black-body radiation spectrum. This lead to the derivation of Planck's Law, which can predict black-body emissions by matching points on the curve. Each point was then multiplied by a constant, ***h***, which is now known as Planck's constant. However, the value for this constant was not confirmed yet. Planck found by testing his constant in the formula for spectral radiance that there were many different solutions for the theoretical oscillators that ended in different values of entropy.

Fig 3. Formulas for spectral radiance where $\boldsymbol{k_B}$ is the Boltzmann constant, $\boldsymbol{h}$ is Plank's constant, $\boldsymbol{c}$ is the speed of light, $\boldsymbol{T}$ is the temperature, $\boldsymbol{v}$ is the frequency, $\boldsymbol{\lambda}$ is the wavelength, and $\boldsymbol{B_v}$ is the body.

$$B_\nu(\nu,T)=\frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{k_BT}}-1}$$

$$B_\lambda(\lambda,T)=\frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_BT}}-1}$$

It wasn't until he switched gears and interpreted the vibrational energy as a distinct measure of an integral number of finite equal parts. Planck was able to derive that the energy must be proportional to the frequency in some way. He correlated the two with his constant stating $\boldsymbol{E = hf}$. Where E is the energy and $\boldsymbol{f}$ is the frequency. With this, Planck was able to calculate the value of his new constant using data previously recorded in black-body experimental data. The result at the time was 6.55 x 10^{-34} Joules x Seconds, and was found to have a 1.2% offset to the value used today, which is 6.626 x 10^{-34} J·s.

Where is Planck's Constant Used

Today, Planck's constant is used in many different physics based applications. The main use it the correlation between the energy that a photon carries and the frequency of the electromagnetic wave it emits. One example of this application is the photoelectric effect from the Planck-Einstein relation, $\boldsymbol{E = h \cdot f}$. The constant in this case was used as a proportionality constant between the frequency of incident light and the kinetic energy of photoelectrons. Another important application for this constant in physics is through the uncertainty principle. In this case, we have $\boldsymbol{\Delta x \cdot \Delta p \geq h}$, where $\boldsymbol{\Delta p}$ is the uncertainty in momentum and $\boldsymbol{\Delta x}$ is the uncertainty in position. This principle states that it is impossible to measure something without disrupting that thing and affecting the measurement. For a third example of how is Planck's constant applied, we have the Bohr Model of the atom. For this application, Planck's constant is used to calculate the angular momentum, $\boldsymbol{L}$, associated with a certain electron height, $\boldsymbol{n}$. The formula for this is $\boldsymbol{L = n \cdot (h/2\pi)}$, where $\boldsymbol{n}$ is always an integer. The final application I will give is the use of thin constant in finding the wavelength of colored light emitting diodes. This application has the formula $\boldsymbol{\lambda = (h \cdot c)/(V_L \cdot Q_e)}$, where $\boldsymbol{V_L}$ is the voltage drop throughout the light and $\boldsymbol{Q_e}$ is the charge on a single electron.

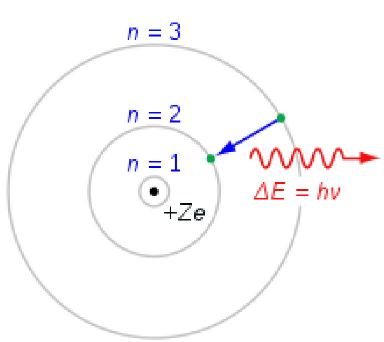


Fig 4. (Left) Bohr's Model of the atom.

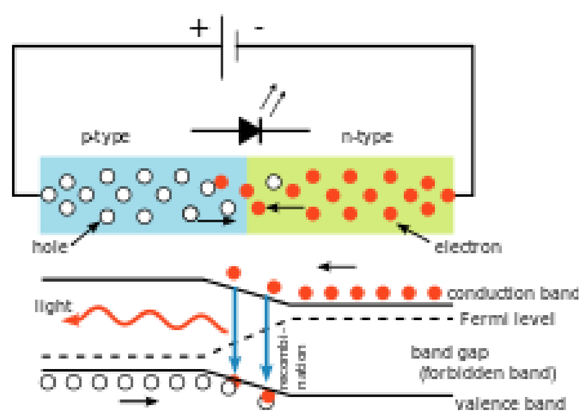


Fig 5. (Right) Illustration of a colored light emitting diode.

Measuring Planck's Constant Today

For this evaluation, I will be attempting to provide a manner in which to measure Planck's constant. This can be done by using colored LED lights, a power supply (a 9 V Battery), two multimeters, and a 1kΩ potentiometer. This first step after gathering the materials is to set them up in a circuit where the ammeter is in series with the LED and the voltmeter in parallel to the LED. (See **Fig 6.**) The purpose of this setup is so that the ammeter can measure the current going through the LED while the voltmeter reads the voltage across it. After this is set up, change the voltage in 60 increments of **0.05V** starting at **0.0V** for each LED and record the current for each. Once all this data is collected, plot it on a graph similar to that of **Fig 7**. With this, calculate the activation voltage (the line where current and voltage move linearly), represented by $\boldsymbol{V_a}$. The activation voltage, in this case, relates directly to the wavelength and is provided by the maker of the LED as they are designed to produce different colors at different voltages. In this case, to get red (λ = 623nm), the activation voltage is 1.78 V. For blue (λ = 586 nm) V_a must be 1.90 V. Green (λ = 567nm) is at 2.00V and, finally, blue (λ = 567nm) at 2.45V.

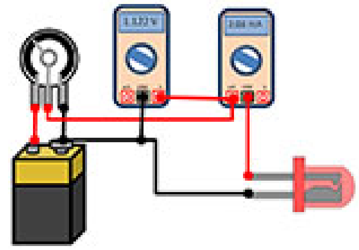
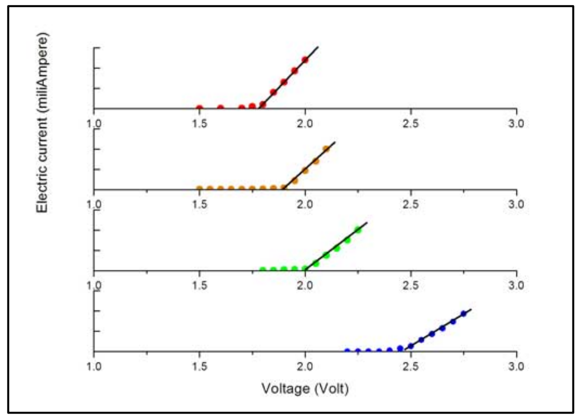


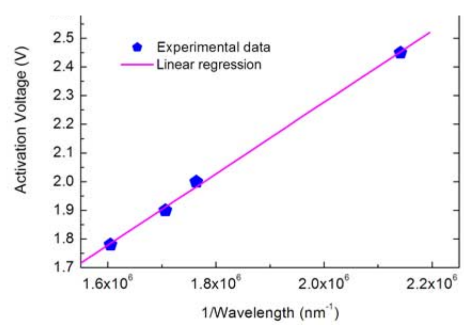
Fig 6. (Above) Configuration.

Fig 7. (Below) Electric Current v/s Voltage.



The activation voltage and wavelength can be used to determine the amount of photons emitted, written as E_p. The correlation formula to do this is $\boldsymbol{V_a = \frac{E_p}{e} + \frac{\varphi}{c}}$ where $\boldsymbol{e}$ is the charge of an electron (1.602 x 10^{-19} coulombs) and $\boldsymbol{E_p = hc/\lambda}$. The term ($\boldsymbol{\varphi/c}$) is a constant where $\boldsymbol{\varphi}$ is a constant that cannot be determined without using multiple wavelengths. Since $\boldsymbol{\varphi}$ is universal to this experiment, it can be pulled out. We can rearrange this formula to make it so that $\boldsymbol{V_a}$ is a function of $\boldsymbol{\lambda}$. This would be the result; $\boldsymbol{V_a = \frac{hc}{e}\left(\frac{1}{\lambda}\right) + \frac{\varphi}{c}}$. This make a graph of V_a against (1/λ) giving a straight line with the gradient of *hc/e*. (See **Fig 8**.) Using linear regression, we find that the slope is 1.24811 x 10^{-6} V·m (Volt Meters). For this, the slope is equal to *hc/e* which can now be solved as a problem. $h = em/c$. To solve this, we just plug in the values we know.

Fig 8. Activation Voltage v/s (1/λ)



$$h = (1.6022x10^{-19}c) \cdot \left(\frac{1.24811x10^{-6}}{2.9979x10^{8}\frac{m}{s}} \right) = 6.6704x10^{-34} J \cdot s$$

This value found has a 0.67% difference from the value used today as Planck's constant, which I believe to be an acceptable value.

References

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Fig 1, 2, 4, and 5 were cited off Google Images.