Homework #7, problem 14.3, 14.4, 14.5, 14.7, 14.8

14.3 ⋆ The density is $\varrho = 0.07\ \text{g/cm}^3$, the length of the tank, $L = 50$ cm, and the mass of an H atom is $m_\text{H} = 1.66 \times 10^{-27}$ kg. The number density (number/volume) is ϱ/m_H, so the target density (number/area) is

$$n_\text{tar} = (\text{number density}) \times L = \frac{\varrho L}{m_\text{H}} = \frac{(0.07 \times 10^3\ \text{kg/m}^3) \times (0.5\ \text{m})}{1.66 \times 10^{-27}\ \text{kg}} \approx 2.1 \times 10^{28}\ \text{atoms/m}^3$$

14.4 ⋆⋆ As in Eq.(14.3), $n_\text{tar} = \varrho t/m$, so

$$N_\text{sc} = N_\text{inc} n_\text{tar} \sigma = 10^9 \times \frac{(8.9 \times 10^3\ \text{kg/m}^3) \times (10^{-5}\ \text{m})}{63.5 \times 1.66 \times 10^{-27}\ \text{kg}} \times (2.0 \times 10^{-28}\ \text{m}^2) = 1.69 \times 10^5\ \text{particles.}$$

14.5 ⋆⋆ The target density is n_tar = (number density) × thickness, where in an ideal gas of nitrogen the number density is $2 \times (6.02 \times 10^{23})/(22.4\ \text{liters}) = 5.38 \times 10^{25}\ \text{particles/m}^3$. (Remember that a mole of gas occupies 22.4 liters and that each N_2 molecule has two atoms.) Thus $n_\text{tar} = 5.38 \times 10^{24}\ \text{particles/m}^2$. Therefore

$$N_\text{sc} = N_\text{inc} n_\text{tar} \sigma = 10^{11} \times (5.38 \times 10^{24}\ \text{m}^{-2}) \times (0.5 \times 10^{-28}\ \text{m}^2) = 2.7 \times 10^7\ \text{particles.}$$

14.7 ⋆ A sphere of radius R at a large distance d subtends a solid angle $\Delta\Omega \approx A/d^2 = \pi R^2/d^2$. For the moon this gives $\Delta\Omega_\text{moon} \approx 6.45 \times 10^{-5}$ sr, and for the sun, $\Delta\Omega_\text{sun} \approx 6.76 \times 10^{-5}$ sr. Because $\Delta\Omega_\text{moon} \approx \Delta\Omega_\text{sun}$, the moon and sun *appear* to be about the same size.

14.8 ⋆ $\Delta\Omega = A/r^2 = (1\ \text{mm}^2)/(10\ \text{mm})^2 = 0.01$ sr.