Homework #11, problems: 47, 48, 56, 57, 62, 74, 75 in chapter 15

15.47 ⋆ Since he had to be approaching the light head-on (or nearly so), $\theta = 0$ and Eq.(15.64) becomes $\omega = \omega_o\sqrt{(1+\beta)/(1-\beta)}$ (as in Problem 1.46). Solving for β we find that

$$\beta = \frac{\omega^2 - \omega_o^2}{\omega^2 + \omega_o^2} = \frac{\lambda_o^2 - \lambda^2}{\lambda_o^2 + \lambda^2} = \frac{65^2 - 53^2}{65^2 + 53^2} = 0.20.$$

His speed had to be about $0.20c$.

15.48 ⋆⋆ **(a)** With $\theta = 90°$, Eq.(15.64) reduces to $\omega = \omega_o/\gamma$. If $\beta = 0.2$, then $1/\gamma = \sqrt{1-\beta^2} = \sqrt{1-0.04} \approx 1 - 0.02$. Therefore the percent shift is -2%.

(b) If the source approaches head-on, the observed frequency is $\omega = \omega_o/\gamma(1-\beta) \approx \omega_o(1-0.02)/(1-0.2) \approx 1.22\omega_o$, and the percent shift is $+22\%$.

15.56 ⋆ **(a)** Since $M_i c^2 + T_i = M_f c^2 + T_f$, we see that $\Delta M c^2 = -\Delta T = -5$ eV. Thus

$$\Delta M = -5 \text{ eV}/c^2 = -5.4 \times 10^{-9} \text{ u}.$$

(b) Since the initial mass of two H_2 and one O_2 molecules is 36 u, the fractional change in mass is $\Delta M/M = -(5.4 \times 10^{-9})/36 = -1.5 \times 10^{-10}$.

(c) Whatever the initial mass, the fractional change will be the same, so, with 10 grams initially, the change will be $\Delta M = -1.5 \times 10^{-9}$ gram. Pretty small!

15.57 ⋆ With the initial atom at rest, conservation of energy implies that $M_i c^2 = M_f c^2 + T_f$, so $T_f = (M_i - M_f)c^2 = (m_{At} - m_{Bi} - m_{He})c^2 = (0.0087 \text{ u})c^2 = 8.1 \text{ MeV} = 1.3 \times 10^{-12}$ J.

15.62 ⋆ $E = T + mc^2 = 13$ MeV. Therefore $pc = \sqrt{E^2 - mc^2} = 5$ MeV or $p = 5$ MeV$/c$, and $\beta = pc/E = 5/13 = 0.38$, so $v = 0.38c$.

15.74 ★★ (a) In the CM frame (the rest frame of the original particle a), the two final particles move with equal and opposite three-momenta and equal energies, $E_{b1} = E_{b2} = \frac{1}{2}m_a c^2 = \frac{5}{4}m_b c^2$. Thus either b particle has three-momentum of magnitude given by

$$|\mathbf{p}_b|c = \sqrt{E_b^2 - (m_b c^2)^2} = \sqrt{\left(\tfrac{5}{4}\right)^2 - 1}\; m_b c^2 = \tfrac{3}{4} m_b c^2$$

and speed $v_b = |\mathbf{p}_b|c^2/E_b = 0.6c$.

(b) The two b particles travel in opposite directions with velocities $\pm 0.6c$ along the x axis of the CM frame, and the CM frame travels at speed $0.5c$ relative to the frame $\mathcal{S}$. Thus the velocities relative to $\mathcal{S}$ are given by the inverse velocity transformation as

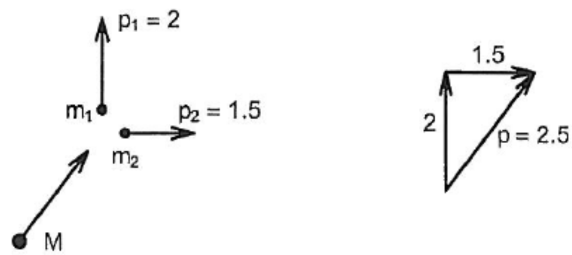
$$v_{b1} = \frac{0.6c + 0.5c}{1 + (0.6)\times(0.5)} = \frac{1.1c}{1.3} = 0.85c \quad \text{and} \quad v_{b2} = \frac{-0.6c + 0.5c}{1 - (0.6)\times(0.5)} = \frac{-0.1c}{0.7} = -0.14c$$

15.75 ★★ Using the useful relation (15.85) we can find the energies of the two final particles

$$E_1 = \sqrt{(p_1 c)^2 + (m_1 c^2)^2} = \sqrt{2^2 + (0.5)^2} = 2.06 \text{ GeV}$$

and

$$E_2 = \sqrt{(p_2 c)^2 + (m_2 c^2)^2} = \sqrt{(1.5)^2 + 1^2} = 1.80 \text{ GeV}$$



By conservation of energy and momentum, the original particle had

$$E = E_1 + E_2 = 3.86 \text{ GeV}$$

and

$$\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2 = 2.5 \text{ GeV}/c, \text{ in the direction shown.}$$

Finally, $M = \sqrt{E^2 - (pc)^2} = \sqrt{3.86^2 - 2.5^2} = 2.95 \text{ GeV}/c^2$, and $\beta = pc/E = 2.5/3.86 = 0.65$.