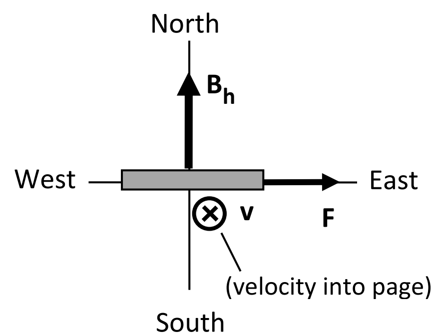
2. ***REASONING*** During its fall, both the length of the bar and its velocity **v** are perpendicular to the horizontal component $\mathbf{B_h}$ of the earth's magnetic field (see the drawing for an overhead view). Therefore, the emf ξ induced across the length *L* of the rod is given by $\xi = vB_h L$ (Equation 22.1), where *v* is the speed of the rod. We will use Equation 22.1 to determine the magnitude B_h of the horizontal component of the earth's magnetic field, and Right-Hand Rule No.1 from Section 21.2 to determine which end of the rod is positive.



Overhead view

SOLUTION

a. Solving $\xi = vB_h L$ (Equation 22.1) for B_h, we find that

$$B_h = \frac{\xi}{vL} = \frac{6.5\times10^{-4}\ \text{V}}{(22\ \text{m/s})(0.80\ \text{m})} = \boxed{3.7\times10^{-5}\ \text{T}}$$

b. Consider a hypothetical positive charge that is free to move inside the falling rod. The bar is falling downward, carrying the positive charge with it, so that the velocity **v** of the charge is downward. In the drawing, which shows the situation as seen from above, downward is into the page. Applying Right-Hand Rule No. 1 to the vectors **v** and $\mathbf{B_h}$, the magnetic force **F** on the charge points to the east. Therefore, positive charges in the rod would accelerate to the east, and negative charges would accelerate to the west. As a result, the $\boxed{\text{east}}$ end of the rod acquires a positive charge.

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4. ***REASONING***

a. The motional emf generated by the moving metal rod depends only on its speed, its length, and the magnitude of the magnetic field (see Equation 22.1). The motional emf does not depend on the resistance in the circuit. Therefore, the emfs for the circuits are the same.

b. According to Equation 20.2, the current *I* is equal to the emf divided by the resistance *R* of the circuit. Since the emfs in the two circuits are the same, the circuit with the smaller resistance has the larger current. Since circuit 1 has one-half the resistance of circuit 2, the current in circuit 1 is twice as large.

c. The power P is $P = \xi^2 / R$ (Equation 20.6c), where ξ is the emf (or voltage) and R is the resistance. The emf produced by the moving bar is directly proportional to its speed (see Equation 22.1). Thus, the bar in circuit 1 produces twice the emf, since it's moving twice as fast. Moreover, the resistance in circuit 1 is half that in circuit 2. As a result, the power delivered to the bulb in circuit 1 is $2^2 / \left(\frac{1}{2}\right) = 8$ times greater than in circuit 2.

SOLUTION

a. The ratio of the emfs is, according to Equation 22.1

$$\frac{\xi_1}{\xi_2} = \frac{vBL}{vBL} = \boxed{1}$$

b. Equation 20.2 states that the current is equal to the emf divided by the resistance. The ratio of the currents is

$$\frac{I_1}{I_2} = \frac{\xi_1/R_1}{\xi_2/R_2} = \frac{R_2}{R_1} = \frac{110\,\Omega}{55\,\Omega} = \boxed{2}$$

c. The power, according to Equation 20.6c, is $P = \xi^2/R$. The motional emf is given by Equation 22.1 as $\xi = vBL$. The ratio of the powers is

$$\frac{P_1}{P_2} = \frac{\dfrac{\xi_1^2}{R_1}}{\dfrac{\xi_2^2}{R_2}} = \left(\frac{\xi_1}{\xi_2}\right)^2 \left(\frac{R_2}{R_1}\right) = \left(\frac{v_1 BL}{v_2 BL}\right)^2 \left(\frac{R_2}{R_1}\right)$$

$$= \left(\frac{v_1}{v_2}\right)^2 \left(\frac{R_2}{R_1}\right) = \left(\frac{2v_2}{v_2}\right)^2 \left(\frac{R_2}{\frac{1}{2}R_2}\right) = \boxed{8}$$

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11. ***REASONING*** According to Equation 22.2, the magnetic flux Φ is the product of the magnitude *B* of the magnetic field, the area *A* of the surface, and the cosine of the angle ϕ between the direction of the magnetic field and the normal to the surface. The area of a circular surface is $A = \pi r^2$, where *r* is the radius.

SOLUTION The magnetic flux Φ through the surface is

$$\Phi = BA\cos\phi = B\left(\pi r^2\right)\cos\phi = (0.078\text{ T})\,\pi\,(0.10\text{ m})^2\cos 25° = \boxed{2.2\times10^{-3}\text{ Wb}}$$

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19. ***REASONING*** According to Faraday's law, as given in Equation 22.3, the magnitude of the emf is $|\xi| = \left|-(1)\frac{\Delta\Phi}{\Delta t}\right|$, where we have set *N* = 1 for a single turn. Since the normal is parallel to the magnetic field, the angle ϕ between the normal and the field is ϕ = 0° when calculating the flux Φ from Equation 22.2: Φ = *BA* cos 0° = *BA*. We will use this expression for the flux in Faraday's law.

SOLUTION Representing the flux as Φ = *BA*, we find that the magnitude of the induced emf is

$$|\xi| = \left|-\frac{\Delta\Phi}{\Delta t}\right| = \left|-\frac{\Delta(BA)}{\Delta t}\right| = \left|-\frac{B\,\Delta A}{\Delta t}\right|$$

In this result we have used the fact that the field magnitude *B* is constant. Rearranging this equation gives

$$\left|\frac{\Delta A}{\Delta t}\right| = \frac{|\xi|}{B} = \frac{2.6\text{ V}}{1.7\text{ T}} = \boxed{1.5\text{ m}^2/\text{s}}$$

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21. SSM ***REASONING*** According to Equation 22.3, the average emf induced in a coil of *N* loops is $\xi = -N\Delta\Phi/\Delta t$.

SOLUTION For the circular coil in question, the flux through a single turn changes by

$$\Delta\Phi = BA\cos 45^\circ - BA\cos 90^\circ = BA\cos 45^\circ$$

during the interval of $\Delta t = 0.010$ s. Therefore, for *N* turns, Faraday's law gives the magnitude of the emf as

$$|\xi| = \left| -N\frac{BA\cos 45^\circ}{\Delta t} \right|$$

Since the loops are circular, the area *A* of each loop is equal to πr^2. Solving for *B*, we have

$$B = \frac{|\xi|\Delta t}{N\pi r^2 \cos 45^\circ} = \frac{(0.065\text{ V})(0.010\text{ s})}{(950)\pi(0.060\text{ m})^2\cos 45^\circ} = \boxed{8.6\times10^{-5}\text{ T}}$$

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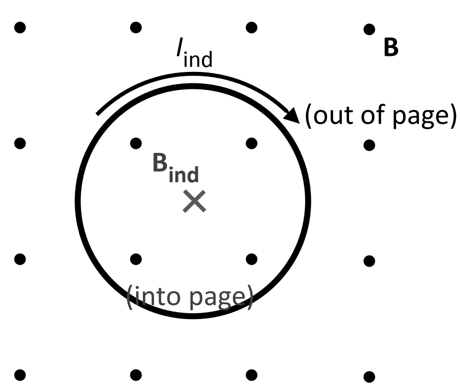
32. ***REASONING*** The external magnetic field is perpendicular to the plane of the horizontal loop, so it must point either upward or downward. We will use Lenz's law to decide whether the external magnetic **B** field points up or down. This law predicts that the direction of the induced magnetic field $\mathbf{B}_{\text{ind}}$ opposes the change in the magnetic flux through the loop due to the external field.

SOLUTION The external magnetic field **B** is increasing in magnitude, so that the magnetic flux through the loop also *increases* with time. In order to oppose the increase in magnetic flux, the induced magnetic field $\mathbf{B}_{\text{ind}}$ must be directed *opposite* to the



external magnetic field **B**. The drawing shows the loop as viewed from above, with an induced current I_{ind} flowing clockwise. According to Right-Hand Rule No. 2 (see Section 21.7), this induced current creates an induced magnetic field $\mathbf{B}_{\text{ind}}$ that is directed *into* the page at the center of the loop (and all other points of the loop's interior). Therefore, the external magnetic field **B** must be directed *out* of the page. Because we are viewing the loop from above, "out of the page" corresponds to $\boxed{\text{upward}}$ toward the viewer.

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36. ***REASONING*** According to Lenz's law, the induced current in the triangular loop flows in such a direction so as to create an induced magnetic field that opposes the original flux change.

SOLUTION

a. As the triangle is crossing the +*y* axis, the magnetic flux down into the plane of the paper is increasing, since the field now begins to penetrate the loop. To offset this increase, an induced magnetic field directed up and out of the plane of the paper is needed. By applying RHR-2 it can be seen that such an induced magnetic field will be created within the loop by a $\boxed{\text{counterclockwise induced current}}$.

b. As the triangle is crossing the –*x* axis, there is no flux change, since all parts of the triangle remain in the magnetic field, which remains constant. Therefore, there is no induced magnetic field, and $\boxed{\text{no induced current appears}}$.

c. As the triangle is crossing the –*y* axis, the magnetic flux down into the plane of the paper is decreasing, since the loop now begins to leave the field region. To offset this decrease, an induced magnetic field directed down and into the plane of the paper is needed. By applying RHR-2 it can be seen that such an induced magnetic field will be created within the loop by a $\boxed{\text{clockwise induced current}}$.

d. As the triangle is crossing the +*x* axis, there is no flux change, since all parts of the triangle remain in the field-free region. Therefore, there is no induced magnetic field, and $\boxed{\text{no induced current appears}}$.

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42. ***REASONING*** The peak emf ξ_0 of a generator is found from $\xi_0 = NAB\omega$ (Equation 22.4), where *N* is the number of turns in the generator coil, *A* is the coil's cross-sectional area, *B* is the magnitude of the uniform magnetic field in the generator, and ω is the angular frequency of rotation of the coil. In terms of the frequency *f* (in Hz), the angular frequency is given by $\omega = 2\pi f$ (Equation 10.6). Substituting Equation 10.6 into Equation 22.4, we obtain

$$\xi_0 = NAB(2\pi f) = 2\pi NABf \tag{1}$$

When the rotational frequency *f* of the coil changes, the peak emf ξ_0 also changes. The quantities *N*, *A*, and *B* remain constant, however, because they depend on how the generator is constructed, not on how rapidly the coil rotates. We know the peak emf of the generator at one frequency, so we will use Equation (1) to determine the peak emf for a different frequency in part (a), and the frequency needed for a different peak emf in part (b).

SOLUTION

a. Solving Equation (1) for the quantities that do not change with frequency, we find that

$$\frac{\xi_0}{f} = \underbrace{2\pi NAB}_{\text{Same for all frequencies}} \tag{2}$$

The peak emf is $\xi_{0,1}$ = 75 V when the frequency is f_1 = 280 Hz. We wish to find the peak emf $\xi_{0,2}$ when the frequency is f_2 = 45 Hz. From Equation (2), we have that

$$\frac{\xi_{0,2}}{f_2} = \underbrace{2\pi NAB}_{\text{Same for all frequencies}} = \frac{\xi_{0,1}}{f_1} \tag{3}$$

Solving Equation (3) for $\xi_{0,2}$, we obtain

$$\xi_{0,2} = \left(\frac{f_2}{f_1}\right)\xi_{0,1} = \left(\frac{45 \text{ Hz}}{280 \text{ Hz}}\right)(75 \text{ V}) = \boxed{12 \text{ V}}$$

b. Letting $\xi_{0,3}$ = 180 V, Equation (2) yields

$$\frac{\xi_{0,3}}{f_3} = \frac{\xi_{0,1}}{f_1} \tag{4}$$

Solving Equation (4) for f_3, we find that

$$f_3 = \left(\frac{\xi_{0,3}}{\xi_{0,1}} \right) f_1 = \left(\frac{180\ \text{V}}{75\ \text{V}} \right) (280\ \text{Hz}) = \boxed{670\ \text{Hz}}$$
